Composite Particles in Quantum Mechanics

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Abstract In relativistic quantum field theory, a clear distinction between elementary and composite particles does not always exist, and entities that are traditionally called composite particles (like mesons) are not necessarily "made of" any well-defined number of constituent particles. This article considers a strictly nonrelativistic model that does have a clear distinction between elementary and composite particles, and its composite particles do have well-defined constituents. Understanding the composite-particle phenomenon in this easier setting is an important step toward the study of particles in relativistic quantum field theory.

Contents

1 Context

Article 41522 contructed a nonrelativistic model of N elementary particles, all of different species. This article uses the $N = 2$ version of that model to introduce the composite-particle phenomenon. This section briefly reviews the definition of the model for arbitrary N.

The number of dimensions of space will be denoted D. This article uses natural units in which Planck's constant is $\hbar = 1$. An element of the Hilbert space is represented by a complex-valued function $\psi(\mathbf{x}_1, ..., \mathbf{x}_N)$, where each $\mathbf{x}_n$ is a list of D real variables. The unitary time translation operators

$$U(t) = \exp(-iHt) \tag{1}$$

are generated by the hamiltonian $H = T + V$, with T and V defined by

$$T\psi(\mathbf{x}_1, ..., \mathbf{x}_N) \equiv \sum_n \frac{-\nabla_n^2}{2m_n}\psi(\mathbf{x}_1, ..., \mathbf{x}_N) \tag{2}$$

$$V\psi(\mathbf{x}_1, ..., \mathbf{x}_N) \equiv \sum_{j<k} V_{jk}(\mathbf{x}_j - \mathbf{x}_k)\psi(\mathbf{x}_1, ..., \mathbf{x}_N). \tag{3}$$

The symbol ∇_n denotes the gradient with respect to $\mathbf{x}_n$. The parameter m_n is the mass of one particle of the nth species, and the function V_{jk} defines the interaction between species j and k.

In the Heisenberg picture, the model's basic observables are represented by projection operators $Q_n(R, t)$. These projection operators are defined by

$$Q_n(R, t) = U^{-1}(t)Q_n(R, 0)U(t) \tag{4}$$

with

$$Q_n(R, 0)\psi(\mathbf{x}_1, ..., \mathbf{x}_N) \equiv \begin{cases} \psi(\mathbf{x}_1, ..., \mathbf{x}_N) & \text{if } \mathbf{x}_n \in R \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

A measurement of this observable has two possible outcomes: a particle of the nth species either is or is not present in the region R at time t.

2 Perspective and approach

The model's basic observables represent localized detectors, each sensitive to one of the N elementary species. The assertion that the observable $Q_n(R,t)$ detects *something* in the region R at time t is an important input to the model's definition, but the assertion that it detects an *individual particle* is not really needed, because it can be inferred by studying how the thing that $Q_n(R,t)$ detects behaves over time.

The set of observables localized in R at time t includes the basic observables $Q_n(R,t)$, but it also includes others, as defined in article 41522 and reviewed in section 10. For some of these observables, the thing they detect may act like a particle even though it is not among the species defined by the basic observables $Q_n(R,t)$. In those cases, we can interpret the thing they detect as a composite particle.

In general, determining that something acts like a particle may involve difficult calculations, but this article considers a model in which it's relatively easy, namely the two-species ($N=2$) version of the model reviewed in section 1. Depending on the choice of the interaction term V_{12}, the model may have states whose time-dependence in the Schrödinger picture has the form

$$\psi(\mathbf{x}_1,\mathbf{x}_2,t) = f(\mathbf{x}_{\text{com}},t)g(\mathbf{x}_1-\mathbf{x}_2)$$

with

$$\mathbf{x}_{\text{com}} \equiv \frac{m_1\mathbf{x}_1+m_2\mathbf{x}_2}{m_1+m_2}.$$

(The subscript "com" stands for center-of-mass.) The function $f(\mathbf{x}_{\text{com}},t)$ satisfies the Schrödinger equation for a single free particle with mass $M\equiv m_1+m_2$, and the time-independent factor $g(\mathbf{x}_1-\mathbf{x}_2)$ is negligible whenever $|\mathbf{x}_1-\mathbf{x}_2|$ is larger than some small finite size. From this, we can infer that this state represents a composite particle, and we can construct local observables that detect it.[1]

[1] This requires a minor compromise: either the observables are only approximately local (to a good approximation), or they detect the composite particle with imperfect (but nearly perfect) reliability. A similar compromise is necessary for *all* particles in relativistic quantum field theory.

3 Separating the center of mass: preview

The next section shows that for any N, the hamiltonian H defined in section 1 can be rewritten as[2]

$$H = H_{\text{com}} + H_{\text{rel}} \tag{6}$$

with

$$H_{\text{com}} = \frac{-1}{2M}\nabla^2_{\text{com}} \qquad\qquad H_{\text{rel}} = \frac{-1}{2M}\sum_{j<k}\nabla^2_{j,k} + V$$

and

$$M \equiv \sum_n m_n \qquad\qquad \nabla_{\text{com}} = \sum_n \nabla_n \qquad\qquad \nabla_{j,k} \equiv \frac{m_j\nabla_k - m_k\nabla_j}{\sqrt{m_j m_k}}.$$

This is useful because the operators H_{com} and H_{rel} commute with each other,[3] so the unitary time evolution operator (1) factorizes:

$$U(t) = U_{\text{com}}(t)U_{\text{rel}}(t)$$

with

$$U_{\text{com}}(t) \equiv \exp(-iH_{\text{com}}\,t) \qquad\qquad U_{\text{rel}}(t) \equiv \exp(-iH_{\text{rel}}\,t).$$

The Hilbert space does not have any state-vectors that are eigenstates of H_{com}, but it may have state-vectors that are eigenstates of H_{rel}:

$$H_{\text{rel}}|\psi\rangle \propto |\psi\rangle.$$

In this case, all of the time-dependence (excluding an irrelevant overall factor) comes from H_{com}, which has the same form as the hamiltonian for a single free particle with mass M. We will see that such a state can be interpreted as a composite particle.

[2]The subscripts "com" and "rel" stand for *center-of-mass* and *relative*, respectively.

[3]This relies on the fact that V is invariant under translations in space. The generator of translations in space is proportional to ∇_{com}.

4 Separating the center of mass: derivation

This section shows that the hamiltonian H defined in section 1 can be rewritten as shown in section 3. Start by using the definitions of ∇_{com} and $\nabla_{j,k}$ to get

$$\nabla^2_{\text{com}} = \sum_n \nabla^2_n + \sum_{j\neq k} \nabla_j \nabla_k$$
$$\frac{1}{2}\sum_{j\neq k} \nabla^2_{j,k} = \sum_{j\neq k} \frac{m_j}{m_k}\nabla^2_k - \sum_{j\neq k} \nabla_j \nabla_k.$$

(Each sum over $j \neq k$ is a sum over all pairs j, k for which $j \neq k$.) Combine these to get

$$\begin{aligned}
\nabla^2_{\text{com}} + \sum_{j<k} \nabla^2_{j,k} &= \nabla^2_{\text{com}} + \frac{1}{2}\sum_{j\neq k} \nabla^2_{j,k} \\
&= \sum_n \nabla^2_n + \sum_{j\neq k} \frac{m_j}{m_k}\nabla^2_k \\
&= \sum_n \nabla^2_n + \sum_k \frac{\nabla^2_k}{m_k}(M - m_k) \\
&= M \sum_k \frac{\nabla^2_k}{m_k}.
\end{aligned}$$

Multiply both sides by $-1/2M$ and then add V to both sides to get

$$H_{\text{com}} + H_{\text{rel}} = H,$$

which is equation (6).

5 Factorizing the Schrödinger equation

Let $\mathbf{x}_{\text{com}}$ denote the center-of-mass coordinate

$$\mathbf{x}_{\text{com}} \equiv \frac{1}{M}\sum_n m_n \mathbf{x}_n. \tag{5}$$

The differential operators ∇_{com} and $\nabla_{j,k}$ satisfy[4]

$$\nabla_{\text{com}} f(\mathbf{x}_j - \mathbf{x}_{\text{com}}) = 0 \qquad\qquad \nabla_{j,k} f(\mathbf{x}_{\text{com}}) = 0 \tag{7}$$

for all functions f. Also, the interaction term V can be written entirely in terms of the combinations $\mathbf{x}_j - \mathbf{x}_{\text{com}}$, so the Schrödinger equation

$$i\frac{\partial}{\partial t}\psi(\mathbf{x}_1, ..., \mathbf{x}_N, t) = H\psi(\mathbf{x}_1, ..., \mathbf{x}_N, t) \tag{8}$$

is satisfied by

$$\psi(\mathbf{x}_1, ..., \mathbf{x}_N, t) = \psi_{\text{com}}(\mathbf{x}_{\text{com}}, t)\psi_{\text{rel}}(\mathbf{x}_1 - \mathbf{x}_{\text{com}}, ..., \mathbf{x}_N - \mathbf{x}_{\text{com}}, t) \tag{9}$$

whenever the factors ψ_{com} and ψ_{rel} satisfy

$$i\frac{\partial}{\partial t}\psi_{\text{com}} = H_{\text{com}}\psi_{\text{com}} \qquad\qquad i\frac{\partial}{\partial t}\psi_{\text{rel}} = H_{\text{rel}}\psi_{\text{rel}}. \tag{10}$$

The next section specializes this to the two-species model ($N = 2$).

[4]Here's a quick way to deduce the first equation: use the fact that ∇_{com} generates translations in space and that $f(\mathbf{x}_j - \mathbf{x}_{\text{com}})$ is invariant under translations in space.

6 Specialization to two species

Now specialize to the two-species model ($N = 2$). In this case, the Schrödinger equation (8) is

$$i\frac{\partial}{\partial t}\psi(\mathbf{x}_1,\mathbf{x}_2,t) = \left(-\frac{\nabla_1^2}{2m_1} - \frac{\nabla_2^2}{2m_2} + V(\mathbf{x}_1-\mathbf{x}_2)\right)\psi(\mathbf{x}_1,\mathbf{x}_2,t), \tag{11}$$

writing $V(\mathbf{x})$ instead of $V_{12}(\mathbf{x})$ because those subscripts are no longer needed. Use the identities

$$\mathbf{x}_1 - \mathbf{x}_{\text{com}} = \frac{m_2}{M}(\mathbf{x}_1-\mathbf{x}_2) \qquad\qquad \mathbf{x}_2 - \mathbf{x}_{\text{com}} = \frac{m_1}{M}(\mathbf{x}_2-\mathbf{x}_1),$$

to see that the factor ψ_{rel} in (9) now depends only on the combination $\mathbf{x}_1-\mathbf{x}_2$, so (9) may be written

$$\psi(\mathbf{x}_1,\mathbf{x}_2,t) = \psi_{\text{com}}(\mathbf{x}_{\text{com}},t)g(\mathbf{x}_1-\mathbf{x}_2,t). \tag{12}$$

The identities

$$\nabla_{\text{com}}f(\mathbf{x}_{\text{com}}) = \big[\nabla f(\mathbf{x})\big]_{\mathbf{x}=\mathbf{x}_{\text{com}}} \qquad\qquad \nabla_{1,2}f(\mathbf{x}_1-\mathbf{x}_2) = \frac{-M}{\sqrt{m_1m_2}}\big[\nabla f(\mathbf{x})\big]_{\mathbf{x}=\mathbf{x}_1-\mathbf{x}_2}$$
$$\nabla_{\text{com}}f(\mathbf{x}_1-\mathbf{x}_2) = 0 \qquad\qquad \nabla_{1,2}f(\mathbf{x}_{\text{com}}) = 0$$

hold for all functions f, so ∇_{com} and $\nabla_{1,2}$ are proportional to the gradients with respect to the variables $\mathbf{x}_{\text{com}}$ and $\mathbf{x}_1-\mathbf{x}_2$, respectively. Altogether, this shows that if the factors on the right-hand side of (12) satisfy the one-point Schrödinger equations

$$i\frac{\partial}{\partial t}\psi_{\text{com}}(\mathbf{x},t) = \frac{-\nabla^2}{2M}\psi_{\text{com}}(\mathbf{x},t) \tag{13}$$
$$i\frac{\partial}{\partial t}g(\mathbf{x},t) = \left(\frac{-\nabla^2}{2m} + V(\mathbf{x})\right)g(\mathbf{x},t) \tag{14}$$

with $m \equiv m_1m_2/M$, then their product (12) automatically satisfies the $N = 2$ version of the Schrödinger equation (8).

7 Composite particles

At any given time t, the function (12) must have finite norm in order to represent an element of the Hilbert space. This implies that both factors must have finite norm:

$$\int d^D x\, \big|\psi_{\text{com}}(\mathbf{x},t)\big|^2 < \infty \qquad\qquad \int d^D x\, \big|g(\mathbf{x},t)\big|^2 < \infty.$$

Depending on the form of the potential term V, the equation

$$\left(\frac{-\nabla^2}{2m} + V(\mathbf{x})\right) g(\mathbf{x}) \propto g(\mathbf{x}) \tag{15}$$

may or may not have any solutions with finite norm.[5] Suppose that it does, and let $g(\mathbf{x})$ be one such solution. Then equation (14) has a solution of the form

$$g(\mathbf{x},t) = e^{-iEt} g(\mathbf{x}),$$

where E is the proportionality factor in (15). These are called **bound states** with **internal energy** E. For such a solution, the function (12) has the form that was promised in section 1, and equations (13)-(14) show that it satisfies

$$i\frac{\partial}{\partial t}\psi(\mathbf{x}_1,\mathbf{x}_2,t) = \left(\frac{\mathbf{P}^2}{2M} + E\right)\psi(\mathbf{x}_1,\mathbf{x}_2,t), \tag{16}$$

where

$$\mathbf{P} \equiv -i\sum_n \nabla_n$$

is the total momentum operator, the generator of translations in space. Equation (16) looks like the Schrödinger equation for a single free particle of mass M (article 20554), with an inconsequential constant term added to the hamiltonian. In this way, any solution of equation (15) gives a composite particle.[6] The time-independent function $g(\mathbf{x}_1 - \mathbf{x}_2)$ describes its internal structure.

[5] If $V = 0$, then it doesn't.

[6] It could be a superposition of two or more different species of composite particle. Different species can be distinguished from each other by considering spacetime symmetries, as explained in section 9.

8 Example

Consider three-dimensional space ($D = 3$) and an interaction term of the form

$$V(\mathbf{x}_1 - \mathbf{x}_2) = -\frac{\lambda}{|\mathbf{x}_1 - \mathbf{x}_2|} \tag{17}$$

with a positive constant λ. This is an attractive Coulomb interaction. In this case, equation (15) has a bound state solution of the form[7]

$$g(\mathbf{x}) \propto e^{-\lambda m|\mathbf{x}|}$$

with internal energy $E = -m\lambda^2/2$, so the functions

$$\psi(\mathbf{x}_1, \mathbf{x}_2, t) \propto \psi_{\text{com}}(\mathbf{x}_{\text{com}},\, t)e^{-\lambda m|\mathbf{x}_1 - \mathbf{x}_2|} \tag{18}$$

satisfy the two-species version of the Schrödinger equation (8). This is an example of a composite particle. The factor

$$e^{-\lambda m|\mathbf{x}_1 - \mathbf{x}_2|} \tag{19}$$

describes its internal structure. The most important features of (19) are:

- It is independent of time. The noun **orbital** is often used for such a static configuration. (Not *orbit*, which implies time-dependence.)

- It cannot be factorized into a product $g_1(\mathbf{x}_1)g_2(\mathbf{x}_2)$. The constituent particles are **entangled** with each other.

- It is negligible for large relative distances $|\mathbf{x}_1 - \mathbf{x}_2| \gg 1/(\lambda m)$.

This example has a famous application: if we take the two original species to be an electron and a proton, respectively (ignoring their spins), then the solution (18) corresponds to an isolated hydrogen atom.[8] In this model, the electron and proton are elementary particles, and the hydrogen atom is a composite particle.

[7]This is highlighted in many introductions to quantum mechanics, including equation (4.80) in Griffiths (1995).

[8]This is only an approximation, partly because the model ignores the dynamics of the quantum electromagnetic field, which is what would allow transitions between the different internal energy levels of atoms.

9 Using spacetime symmetry to delineate species

Article 41522 described some spacetime symmetries of the N-species model, including translations, rotations, and the nonrelativistic version of boosts. These transformations generate the **Galilei group**, the connected part of the group of spacetime symmetries of any strictly nonrelativistic model.

A given model may have composite particles of different species with the same internal energy.[9] In that case, the different species can be distinguished from each other by considering the group of spacetime symmetries.[10] For any given internal energy E, the set of all composite-particle states with that internal energy is a subspace of the Hilbert space. This subspace is self-contained under the action of the Galilei group because H_{rel} is invariant under the Galilei group (article 41522). However, this subspace may or may not be **reducible**: it may or may not contain a smaller subspace (other than the trivial zero-dimensional subspace) that is self-contained under the action of the Galilei group. If it doesn't contain any such subspace, then it's called **irreducible**. An irreducible subspace represents a single species. This is a precise way of expressing the intuition that states representing the same species can be mixed with each other by spacetime symmetries, and states representing different species cannot.

[9]The model defined by (17) in 3d space has composite-particle solutions with the same internal energies but different internal angular momenta, as reviewed in most applications of this model to the hydrogen atom. Example: it has two composite-particle species with the internal energy $E = -m\lambda^2/8$, one with spin zero (the function $g(\mathbf{x})$ is invariant under rotations) and one with spin $\hbar$ (with functions $g(\mathbf{x})$ that transform like the components of a spatial vector under rotations). Griffiths (1995) mentions this at the bottom of page 138.

[10]In the present model, this is the Galilei group. In a relativistic model, this would be the Poincaré group.

10 Detection observables for composite particles

This section explains how to define detection observables for a composite particle, as promised in section 2. This requires a minor compromise (footnote 1 in section 2): the observables cannot be both perfectly reliable and perfectly localized within any bounded region of space. This section constructs one example that is perfectly reliable but not perfectly localized, and one that is perfectly localized but not perfectly reliable.

In this model, the rule for determining if/where/when an observable is localized can be expressed like this (article 41522): for an observable A to qualify as being localized in region R at time t, it must satisfy

$$AQ_1(\overline{R},t)Q_2(\overline{R},t)|\psi\rangle \propto Q_1(\overline{R},t)Q_2(\overline{R},t)|\psi\rangle \tag{20}$$

for all $|\psi\rangle$, where $\overline{R}$ is the complement of R (the largest region that does not intersect R). The observables $Q_n(R,t)$ themselves clearly satisfy this rule: for them, the right-hand side of equation (20) is zero.

To construct observables that detect composite particles, first consider the projection operators $Q_{\text{com}}(R,t)$ defined by

$$Q_{\text{com}}(R,t) = U^{-1}(t)Q_{\text{com}}(R,0)U(t) \tag{21}$$

with

$$Q_{\text{com}}(R,0)\psi(\mathbf{x}_1,\mathbf{x}_2) \equiv \begin{cases} \psi(\mathbf{x}_1,\mathbf{x}_2) & \text{if } \mathbf{x}_{\text{com}} \in R \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

These projection operators are *not* local, not even approximately: they don't even come close to satisfying the condition (20).[11] They are also non-selective: they detect *any* configuration, not just composite particles of the desired species. These

[11] To see this, let $b(\mathbf{x})$ be a **bump function** whose support is contained within R, and consider the function $\psi(\mathbf{x}_1,\mathbf{x}_2) = b(\mathbf{x}_1+\mathbf{c})b(\mathbf{x}_2-\mathbf{c}) + b(\mathbf{x}_1+\mathbf{c})b(\mathbf{x}_2+\mathbf{c})$ for a displacement $\pm\mathbf{c}$ that moves the bump function's support completely outside of R. If $m_1 = m_2$, then $Q_{\text{com}}(R,0)\psi(\mathbf{x}_1,\mathbf{x}_2) = b(\mathbf{x}_1+\mathbf{c})b(\mathbf{x}_2-\mathbf{c})$, which is not proportional to $\psi(\mathbf{x}_1,\mathbf{x}_2)$. This violates the condition (20) because $\psi(\mathbf{x}_1,\mathbf{x}_2) = Q_1(\overline{R},0)Q_2(\overline{R},0)\psi(\mathbf{x}_1,\mathbf{x}_2)$.

projection operators are not the observables we want, but they can be used as an ingredient to help construct the observables we want.

To construct a perfectly reliable detection observable for a given species of composite particle, start with the fact that the states representing that species constitute a closed subspace of the Hilbert space, consisting of all functions of the form

$$\psi(\mathbf{x}_1, \mathbf{x}_2) = \psi_{\text{com}}(\mathbf{x}_{\text{com}})g(\mathbf{x}_1 - \mathbf{x}_2) \tag{23}$$

where the factor ψ_{com} is arbitrary and the factor g runs over whichever solution(s) of (15) correspond to the given species.[12] Let P be the projection operator that projects onto that subspace. By construction, P detects the desired species with perfect reliability, but P is not a local operator, not even approximately: it doesn't even come close to satisfying the condition 20.[13] However, consider the product

$$P_{\text{com}}(R, t) \equiv Q_{\text{com}}(R_\epsilon, t)P \tag{24}$$

where R_ϵ is obtained from R by shrinking R in every dimension by some distance ϵ that is much larger than the characteristic width of the bound-state functions $g(\mathbf{x})$ that define P (sections 6-7).[14] For any given R and t, the projection operators $Q_{\text{com}}(R, t)$ and P commute with each other, so their product (24) is another projection operator. The product (24) still doesn't quite satisfy the condition 20, but it does come close:

$$P_{\text{com}}(R, t)\, Q_1(\overline{R}, t)Q_2(\overline{R}, t)|\psi\rangle \approx 0,$$

where "≈ 0" means that the norm of the left-hand side is much less than the norm of $Q_1(\overline{R}, t)Q_2(\overline{R}, t)|\psi\rangle$. The right-hand side is not exactly zero, because for realistic choices of $V(\mathbf{x})$, any solution of (15) has "tails" extending to arbitrarily large values

[12] A composite particle with spin zero corresponds to just one such solution, which is invariant under rotations. In 3-dimensional space, a composite particle with spin $\ell \geq 1$ corresponds to $2\ell + 1$ linearly independent solutions (and all of their linear combinations), which mix with each other under rotations.

[13] This is clear from the fact that the subspace onto which P projects is defined in terms of a function $g(\mathbf{x}_1 - \mathbf{x}_2)$ that is invariant under translations.

[14] For the example in section 8, this means $\epsilon \gg 1/\lambda m$.

of $|\mathbf{x}|$. However, by taking ϵ to be much larger than the characteristic width of $g(\mathbf{x})$, the factor $Q_{\text{com}}(R_\epsilon, t)$ ensures that these "tails" have fallen to a negligible magnitude by the time they reach into $\overline{R}$. This shows that the observable (24) is *approximately* localized in R at time t. For any state of the form

$$\psi(\mathbf{x}_1, \mathbf{x}_2, t) = \psi_{\text{com}}(\mathbf{x}_{\text{com}}, t) g(\mathbf{x}_1 - \mathbf{x}_2) \tag{25}$$

in the subspace selected by P, the probability of the outcome "the composite particle is (approximately) in R" when the observable (24) is measured is

$$\begin{aligned} p &= \frac{\int (d^D x)_1\, (d^D x)_2\ \psi^*(\mathbf{x}_1, \mathbf{x}_2, t)\, P_{\text{com}}(R, t)\, \psi(\mathbf{x}_1, \mathbf{x}_2, t)}{\int d^D x_1\, d^D x_2\ \big|\psi(\mathbf{x}_1, \mathbf{x}_2, t)\big|^2} \\ &= \frac{\int_{\mathbf{x}\in R_\epsilon} d^D x\ \big|\psi_{\text{com}}(\mathbf{x}, t)\big|^2}{\int d^D x\ \big|\psi_{\text{com}}(\mathbf{x}, t)\big|^2}. \end{aligned} \tag{26}$$

Except for the small margin of size ϵ, this is just like the detection probability for a single particle governed by the one-point Schrödinger equation (13).

We can modify the preceding observable to be perfectly localized, at the expense of perfect reliability. For each function $g(\mathbf{x})$ in equation (23) corresponding to the composite-particle species of interest, define a truncated function $\tilde{g}(\mathbf{x})$ that is equal to $g(\mathbf{x})$ for $|\mathbf{x}| < \epsilon$ and equal to zero for $|\mathbf{x}| > \epsilon$. The truncated function $\tilde{g}(\mathbf{x})$ will be a good approximation to the exact solution $g(\mathbf{x})$ if ϵ is large enough. Now define $\tilde{P}$ to be the projection onto the subspace of functions of the form (23) with each g replaced by its truncated version $\tilde{g}$. Then the observable

$$\tilde{P}_{\text{com}}(R, t) \equiv Q_{\text{com}}(R_\epsilon, t)\tilde{P} \tag{27}$$

is perfectly localized within R:

$$\tilde{P}_{\text{com}}(R, t)\, Q_1(\overline{R}, t) Q_2(\overline{R}, t)|\psi\rangle = 0,$$

However, this new observable is not a perfectly reliable detector of the given composite-particle species, because $\tilde{P}$ and P project onto slightly different subspaces. When the observable (27) is measured starting with any state (25) in the subspace corresponding to the given species, the detection probability is like (26), but with an extra factor:

$$\tilde{p} = \frac{\int (d^Dx)_1\,(d^Dx)_2\;\psi^*(\mathbf{x}_1,\mathbf{x}_2,t)\,\tilde{P}_{\text{com}}(R,t)\,\psi(\mathbf{x}_1,\mathbf{x}_2,t)}{\int d^Dx_1\,d^Dx_2\;\big|\psi(\mathbf{x}_1,\mathbf{x}_2,t)\big|^2}$$

$$= \frac{\int_{\mathbf{x}\in R_\epsilon} d^Dx\;\big|\psi_{\text{com}}(\mathbf{x},t)\big|^2}{\int d^Dx\;\big|\psi_{\text{com}}(\mathbf{x},t)\big|^2} \tag{28}$$

$$\times\sum_k \frac{\big|\int d^Dx\;g^*(\mathbf{x})\tilde{g}_k(\mathbf{x})\big|^2}{\Big(\int d^Dx\;\big|g(\mathbf{x})\big|^2\Big)\Big(\int d^Dx\;\big|\tilde{g}_k(\mathbf{x})\big|^2\Big)}, \tag{29}$$

where $\tilde{g}_k$ is a set of mutually orthogonal functions defining the subspace selected by $\tilde{P}$. The last factor is only slightly less than 1,[15] so this is almost the same as the probability (26) obtained using the perfectly-reliable detector.

This section was long, but the message is simple: we can construct observables that detect composite particles of a given species in a given region of space at a given time, to a good approximation. Equation (16) says that the thing these observables detect behaves like a single free particle. Altogether, this justifies interpreting a state of the form (12) as a composite particle whenever g satisfies (15).

[15] The corresponding factor in equation (26) is not shown there because it's exactly equal to 1.

11 Detecting a constitutent

What happens if we measure one of the original detection observables $Q_n(R,t)$, for one of the elementary particle species, starting with a composite-particle state? We can study this qualitatively using what article 03431 calls the artificial approach – applying the state-replacement rule directly to the observable $Q_n(R,t)$, without trying to treat the measurement as a physical process.

Consider a state of the form (25), where $g(\mathbf{x})$ satisfies (15). In the artificial approach, after a measurement of the observable $Q_n(R,t)$, the state is replaced with either $Q_n(R,t)\psi(\mathbf{x}_1,\mathbf{x}_2,t)$ or $\big(1-Q_n(R,t)\big)\psi(\mathbf{x}_1,\mathbf{x}_2,t)$, depending on the outcome of the measurement. Either way, the resulting state may be written as a sum of two terms: one that has the form (25) with the same bound-state function g as before, and one that does not have that form. The implications for the composite particle depend on the resolution of the measurement:

- If the region R is much larger than the characteristic width of the bound-state function $g(\mathbf{x})$, then the second term will be negligible:[16] the composite particle will almost certainly remain intact.

- If the region R is much smaller than the characteristic width of the bound-state function $g(\mathbf{x})$, then the second term will be much larger than the first term: the composite particle will almost certainly be severely disrupted by the measurement, maybe even broken apart.

We could quantify this more precisely using the natural approach, in which the measurement is treated as a physical process, but the conclusion would remain essentially the same: detecting a constituent with resolution much coarser than the size of the bound state tends to leave the composite particle intact, and detecting a constituent with much finer resolution tends to be much more disruptive.

[16]This assumes that the support of the function (25) is not concentrated at the boundary of the region R. The boundary effects are mostly an artifact of the artificial approach, because the projection operator $Q_n(R,t)$ has "sharp edges." We could use a generalized measurement, like the one that was illustrated in article 20554 in the context of a single-particle model, to get a more realistic version of the artificial approach without such a sharp boundary effect.

12 References

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13 References in this series

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