Relationship Between the Stress-Energy Tensors

Randy S

Abstract Article 49705 showed that in flat spacetime, translation invariance implies a conservation law of the form $\partial_a T_C^{ab} = 0$. Article 37501 showed that in general relativity, the equation of motion for the metric field involves a different quantity T_H^{ab} that also satisfies the conservation law $\partial_a T_H^{ab} = 0$ when spacetime is flat. Both T_C^{ab} and T_H^{ab} are called the stress-energy tensor (the subscripts C and H stand for *canonical* and *Hilbert*, respectively), but their definitions are different: they are generally not equal to each other, and they are conserved for what appear to be different reasons. After reviewing those reasons, this article illustrates how an ambiguity in the definition of T_C^{ab} can be exploited to make it "practically" the same as T_H^{ab}.

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1 Why T_C^{ab} is conserved: a quick review

Article 49705 showed that translation symmetry leads to a local conservation law of the form $\partial_a T_C^{ab} = 0$. This section reviews the derivation.

Consider a model whose dynamic entities are fields ϕ_n satisfying the action principle. For any given region R of N-dimensional spacetime, the action is

$$S_R = \int_R d^N x\ L(x),$$

where the lagrangian $L(x)$ is a function of the fields ϕ_n and their first derivatives $\partial_a \phi_n$ at the point x. For any variation $\delta\phi_n$ of the fields, the resulting variation of the action is[1]

$$\begin{aligned}
\delta S_R &= \sum_n \int_R d^N x\ \left[\frac{\delta L}{\delta \phi_n}\delta\phi_n + \frac{\delta L}{\delta\,\partial_a\phi_n}\delta\,\partial_a\phi_n\right] \\
&= \sum_n \int_R d^N x\ \left[\Psi_n\,\delta\phi_n + \partial_a\left(\frac{\delta L}{\delta\,\partial_a\phi_n}\delta\phi_n\right)\right] \qquad (1)
\end{aligned}$$

and

$$\Psi_n \equiv \frac{\delta L}{\delta\phi_n} - \partial_a \frac{\delta L}{\delta\,\partial_a\phi_n}. \qquad (2)$$

A variation $\delta\phi_n$ is called a symmetry if

$$\delta S_R = \int_R d^N x\ \partial_a \Lambda^a \qquad (3)$$

for some $\Lambda^a(x)$ that depends only on the fields and their derivatives at x, regardless of the region R. For such a variation, equations (1) and (3) both hold for all regions R, which implies

$$\sum_n \Psi_n\,\delta\phi_n + \partial_a\left(\sum_n \frac{\delta L}{\delta\,\partial_a\phi_n}\delta\phi_n - \Lambda^a\right) = 0. \qquad (4)$$

[1] A sum over a is implied.

If the fields satisfy their equations of motion $\Psi_n = 0$, then (4) reduces to $\partial_a J^a = 0$ with
$$J^a \equiv \sum_n \frac{\delta L}{\delta\, \partial_a \phi_n} \delta\phi_n - \Lambda^a. \tag{5}$$
This is a (local) conservation law.

Translation symmetry corresponds to variations of the form
$$\delta\phi = \epsilon_b \partial^b \phi$$
with constant ϵ_b. In this case, we can take the quantity Λ^a in equation (3) to be[2] $\Lambda^a = \epsilon^a L$, so equation (4) becomes
$$\sum_n \Psi_n\, \partial^b \phi_n + \partial_a T_C^{ab} = 0 \tag{6}$$
with
$$T_C^{ab} \equiv \sum_n \frac{\delta L}{\delta\, \partial_a \phi_n} \partial^b \phi_n - \eta^{ab} L, \tag{7}$$
where η is the Minkowski metric that was used to raise/lower spacetime indices throughout this derivation. I'll call (7) the **canonical** stress-energy tensor. Equation (6) says that
$$\partial_a T_C^{ab}(x) = 0 \tag{8}$$
whenever the fields satisfy their equations of motion $\Psi_n = 0$.

[2] I worded it this way because equation (3) doesn't specify Λ^a uniquely. It specifies Λ^a only modulo terms k^a that satisfy $\partial_a k^a = 0$ identically, like $k^a = \partial_b A^{ab}$ with $A^{ab} = -A^{ba}$. *Identically* means without using the fields' equations of motion.

2 Why T_C^{ab} is conserved: two key messages

This section highlights two important messages about the preceding derivation.

One important message is that translation invariance doesn't uniquely determine T_C^{ab} itself, because equation (6) is equivalent to

$$\sum_n \Psi_n\, \partial^b \phi_n + \partial_a (T_C^{ab} + k^{ab}) = 0$$

for any k^{ab} that is *identically* conserved, which means that $\partial_a k^{ab} = 0$ for all behaviors of the fields, whether or not they satisfy their equations of motion. Any quantity of the form

$$k^{ab} = \partial_c (K^{cab} - K^{acb}) \tag{9}$$

is identically conserved ($\partial_a k^{ab} = 0$).[3] Translation invariance only determines the stress-energy tensor modulo terms that are identically conserved.

Another important message is the nature of the association between translation invariance and the conservation law. The conservation law holds even if the symmetry that led to (6) is the model's only symmetry, and in this sense the conservation law can be regarded as a consequence of that specific symmetry. In other words, when we say that translation invariance is the reason for the conservation law, we are implicitly considering a whole *family* of models, some that have translation symmetry and some that don't. That's is the real significance of Noether's theorem:[4] it tells us which symmetries should be viewed as the *reason* for the conservation laws. The conservation laws themselves can be verified directly using the fields' equations of motion, without the help of Noether's theorem, but recognizing symmetries as the *reason* for the conservation laws requires the larger context.

[3] To see this, use the fact that the quantity in parentheses is antisymmetric under $a \leftrightarrow c$, together with the fact that partial derivatives ∂_a and ∂_c commute with each other.

[4] Noether proved two important theorems in the same paper. The one highlighted here is Noether's *first* theorem (Kosmann-Schwarzback (2011)), but physicists usually just call it Noether's theorem, like I'm doing here.

3 Why T_H^{ab} is conserved: a quick review

The **Hilbert** stress-energy tensor T_H^{ab} is defined by[5]

$$T_H^{ab} \equiv \frac{-2}{\sqrt{|\det g|}} \frac{\delta S_m}{\delta g_{ab}} \tag{10}$$

where g_{ab} is the metric field and S is the action for the matter fields. (The subscript m on S_m stands for "matter.")[6] Article 37501 shows that T_H^{ab} is conserved in any model that has the key property reviewed below.

The key property is sometimes called **general covariance**, but I'm not sure that name is always used consistently, so I'll explain what I mean by it. Suppose that the model's equations of motion come from the action principle, with an action of the form $S_m = \int d^N x \, \sqrt{|\det g(x)|} \, L(x)$, where $L(x)$ is (a coordinate representation of) a scalar field constructed from various other tensor fields. Tensor fields – including scalar fields – have coordinate-free definitions (article 09894), and any diffeomorphism[7] of the spacetime manifold in which they live induces a corresponding transformation of the fields via pullbacks and pushforwards. In article 00418, I used the word **fieldomorphism** for such a transformation of the model's fields.[8] Using that language, the key result is that T_H^{ab} is conserved in any model whose action is invariant under fieldomorphisms.

I won't review the whole derivation here (that's what article 37501 is for), but I will review the final steps. Consider a fieldomorphism with compact support in spacetime. Article 71500 shows that the effect of an infinitesimal fieldomorphism on the metric field is $\delta g_{ab} = \nabla_a \theta_b + \nabla_b \theta_a$ for some θ_a, where ∇ is the (Levi-Civita) covariant derivative. If the action is invariant under fieldomorphisms, then

[5] To accommodate spinor fields, the definition of T_H^{ab} used in this article needs to be modified. In that case, the symmetry of T_H^{ab} relies on the spinor fields satisfying their equations of motion (`https://physics.stackexchange.com/q/678322`).

[6] S_m excludes terms that only involve the metric field. Such terms don't contribute to the model's equations of motion when the metric field is a prescribed background field.

[7] Article 93875 reviews the definition of **diffeomorphism**.

[8] This name is not standard.

it satisfies the identity

$$\int d^N x \; \left(\sum_n \frac{\delta S_m}{\delta\phi_n(x)}\delta\phi_n(x) + 2\frac{\delta S_m}{\delta g_{ab}(x)}\nabla_a\theta_b(x)\right) = 0 \tag{11}$$

where ϕ_n denotes all of the other matter fields (except the metric) and $\delta\phi_n$ denotes how those fields are affected by the same infinitesimal fieldomorphism that was applied to the metric field. We assumed that the fieldomorphism has compact support in spacetime, so we don't need to restrict the domain of integration like we did in section 1, and then the total-derivative term in equation (3) does not contribute. If the fields ϕ_n satisfy their equations of motion $\delta S_m/\delta\phi_n = 0$, then the identity (11) implies the **covariant conservation law**[9]

$$\nabla_a T_H^{ab} = 0. \tag{12}$$

In flat spacetime, we can choose the coordinate system so that the components g_{ab} of the metric are constant,[10] and in that case (12) reduces to

$$\partial_a T_H^{ab} = 0. \tag{13}$$

[9]This is derived in article 37501.

[10]*Constant* here means independent of the coordinates.

4 (How) are T_C^{ab} and T_H^{ab} related?

In most models, T_C^{ab} and T_H^{ab} are not equal to each other. In particular, sections 5 and 6 show examples of models in which they are not equal to each other in flat spacetime – not even modulo identically-conserved terms. However, at least in those examples, T_C^{ab} and T_H^{ab} are still related to each other in a more subtle way: in flat spacetime, they are equal to each other modulo identically-conserved terms when the fields satisfy their equations of motion.[11] In the real world, classical field theory is only an approximation (quantum field theory is better), but insofar as it is a useful approximation, classical fields always satisfy their equations of motion. In this practical sense, the difference between T_C^{ab} and T_H^{ab} isn't so great after all.

To help explain why T_C^{ab} and T_H^{ab} are related to each other in this way, two general results will be derived:

- Section 7 shows that T_C^{ab} and T_H^{ab} are equal to each other in models with only scalar fields (and the background metric field).

- Section 8 uses the same approach to show that in a model with only abelian gauge fields (together with the background metric field), we can always add an identically-conserved term to T_C^{ab} to make it equal to T_H^{ab} when the fields satisfy their equations of motion.[12]

The same approach can easily be generalized to models that have both scalar fields and gauge fields. This answers the *why* question, at least for this class of models.[13]

Here's a preview: the proof works by considering arbitrary infinitesimal fieldomorphisms, instead of only compactly-supported fieldomorphisms as in section 3. The translations used in section 1 are not compactly supported in spacetime, but they are fieldomorphisms. By considering *arbitrary* infinitesimal fieldomorphisms,

[11] The condition "when the fields satisfy their equations of motion" is often abbreviated **on-shell**.

[12] The proof assumes that the lagrangian is gauge-invariant and depends only on the fields and their first derivatives (not on any higher derivatives), and that it doesn't involve any derivatives of the metric tensor.

[13] In physics, asking for a deeper answer to a *why* question is the same as asking for a more general theorem.

we are encompassing the symmetries that were used in sections 1 and 3 together.[14] This leads to an equation that has features of both of the earlier equations (4) and (11). This one equation implies both that T_C^{ab} is conserved *and* that it is equal to T_H^{ab} modulo identically-conserved terms when the fields satisfy their equations of motion.

To encompass an even larger class of models, a more sophisticated approach might be more satisfying – maybe like the approach used by Forger and Römer (2003) or by Gotay and Marsden (1992). Section 7.4 in Weinberg (1995) presents a weaker version of a more general result: it's weaker because it doesn't directly address how T_C^{ab} and T_H^{ab} are related to each other, but it does address the fact that T_C^{ab} can be made *symmetric* by adding an identically conserved term, and it does this for a more general class of fields. The intuition reviewed in section 4.1.1 of Tong (2009) is also worth mentioning: it's more of a shortcut than a real explanation, but it is general.

[14]Page 3 in Gotay and Marsden (1992) says "nonconstant deformations [fieldomorphisms that are not translations] are what give rise to the 'correction terms' [that relate T_C^{ab} to T_H^{ab}]."

5 Example: the free electromagnetic field

Maxwell's equations for the electromagnetic field by itself may be written

$$\partial_a F^{ab} = 0 \tag{14}$$

with $F_{ab} \equiv \partial_a A_b - \partial_b A_a$, where A is the gauge field. The equation of motion (14) can be derived from an action principle using the lagrangian

$$L = -\frac{1}{4} F^{ab} F_{ab}. \tag{15}$$

In this case, in flat spacetime, the canonical stress-energy tensor (7) turns out to be[15]

$$T_C^{ab} = \frac{1}{4}\eta^{ab} F^{cd} F_{cd} - F^{ac}\partial^b A_c \tag{16}$$

and the Hilbert stress-energy tensor (10) turns out to be

$$T_H^{ab} = \frac{1}{4}\eta^{ab} F^{cd} F_{cd} - F^{ac} F^b{}_c. \tag{17}$$

The quantity

$$k^{ab} \equiv \partial_c (F^{ac} A^b) \tag{18}$$

is identically conserved ($\partial_a k^{ab} = 0$) because F^{ac} is antisymmetric, so Noether's theorem says that $T_C^{ab} + k^{ab}$ is associated with translation symmetry in the same way that T_C^{ab} is. It's not equal to T_H^{ab}, but the difference is

$$T_C^{ab} + k^{ab} - T_H^{ab} = k^{ab} - F^{ac}\partial_c A^b = (\partial_c F^{ac}) A^b,$$

which is zero when the field satisfies its equation of motion (14). This illustrates the relationship between T_C^{ab} and T_H^{ab} that was previewed in section 4. The generalization of this result to non-abelian gauge fields is reviewed by Blaschke *et al* (2016).

[15] Equation (2.4) in Blaschke *et al* (2016)

6 Example: a charged scalar field

Consider the model specified by the lagrangian $L = L_F + L_\varphi$ with L_F given by (15) and

$$L_\varphi = (D^a\varphi)^*(D_a\varphi),$$

where φ is a complex-valued scalar field and $D_a \equiv \partial_a + iA_a$. The equation of motion for the gauge field is Maxwell's equation with a source term:

$$\partial_a F^{ab} = -J^b \tag{19}$$

with

$$J^b \equiv \frac{\delta L_\varphi}{\delta A_b} = -i\varphi^* D^b\varphi + \text{cc}$$

where "cc" stands for the complex conjugate of the preceding term. The equation of motion for the scalar field is $D^a D_a\varphi = 0$, but we won't need this. We will only need the condition $\partial_b J^b = 0$, which can be derived either directly from the scalar field's equation of motion or indirectly (and much more easily) by applying ∂_b to both sides of Maxwell's equation (19) and using the antisymmetry of F^{ab}.[16] In this model, the definitions (7) and (10) give[17]

$$\begin{aligned}
T_C^{ab} &= T_C^{ab}(F) + (D^a\varphi)^*\partial^b\varphi + \text{cc} - g^{ab}L_\varphi \\
T_H^{ab} &= T_H^{ab}(F) + (D^a\varphi)^*D^b\varphi + \text{cc} - g^{ab}L_\varphi
\end{aligned}$$

with $T_C^{ab}(F)$ and $T_H^{ab}(F)$ given by (16) and (17), respectively. If we choose k^{ab} as before (equation (18)), then

$$T_C^{ab} + k^{ab} - T_H^{ab} = (\partial_c F^{ac})A^b - J^a A^b,$$

which is zero when the electromagnetic field satisfies its equation of motion (19).

[16] Article 19253 explains, in general terms, why the same current-conservation law $\partial_b J^b = 0$ can be derived both ways. This is analogous to why the same covariant conservation law for T_H^{ab} can be derived either directly from the matter-fields' equations of motion or from consistency with the metric field's equation of motion in general relativity (article 37501).

[17] In (10), the variation $\delta/\delta g_{ab}$ is defined by temporarily treating all components of g_{ab} independently (without imposing $g_{ab} = g_{ba}$ until afterward), so before calculating the variation we must write $g^{ab}X_aY_b \to g^{ab}(X_aY_b + X_bY_a)/2$ so that the expression is symmetric even if g_{ab} itself is (temporarily) not.

7 General proof for scalar fields

In a model involving only scalar fields together with the background metric field, the two stress-energy tensors T_C^{ab} and T_H^{ab} are equal to each other in flat spacetime, at least when the fields satisfy their equations of motion. Articles 49705 and 11475 showed that T_C^{ab} and T_H^{ab} are equal to each other for a simple class of scalar-field models *without* using the fields' equations of motion. The result derived here is weaker because it uses the fields' equations of motion, but this approach has the virtue of being extensible to other models (section 8) in which T_C^{ab} and T_H^{ab} are not strictly equal to each other.

As previewed in section 4, the proof accounts for (the infinitesimal version of) symmetry under the group of all fieldomorphisms – the full general covariance group. This includes translations, so we need to restrict the domain of integration to a bounded region R (so that the integral expression for δS is well-defined), and then we need to include the $\partial_a\Lambda^a$ term as in equation (3). To handle fieldomorphisms that are not translations, we need to include the $\delta S/\delta g_{ab}$ term as in equation (11), even if we're only interested in flat spacetime, because no metric is invariant under all fieldomorphisms. Assuming that the action is invariant under both types of fieldomorphism leads to the identity[18]

$$\int_R \left(\Psi\,\delta\phi + \partial_a\left(\frac{\delta\hat{L}}{\delta\,\partial_a\phi}\delta\phi - \Lambda^a\right) + 2\frac{\delta S_R}{\delta g_{ab}}\nabla_a\theta_b\right) = 0 \tag{20}$$

with Ψ defined by (2) and

$$\hat{L} \equiv \sqrt{|\det g|}\,L$$

where L is a scalar field constructed from the model's basic scalar fields ϕ and the metric g. For a generic infinitesimal fieldomorphism parameterized by functions $\theta^a(x)$, the variation of a scalar field ϕ is

$$\delta\phi = \theta^a\partial_a\phi. \tag{21}$$

[18]To reduce clutter, I'm omitting the subscript n on the scalar fields.

The variations of other kinds of tensor fields involve additional terms.[19] To get an explicit expression for the quantity Λ^a in equation (3), use

$$\delta S_R = \int_R \delta\hat{L}.$$

The variation of $\hat{L}$ is

$$\delta\hat{L} = \left(\delta\sqrt{|\det g|}\right) L + \sqrt{|\det g|}\,\delta L.$$

The first term can be evaluated using an identity shown in article 11475, and the second term can be evaluated using the fact that $\hat{L}$ is a scalar field, just like the ϕ in equation (21). After calculating the variations, we can specialize to the Minkowski metric (flat spacetime), with the result

$$\delta\hat{L} = (\partial_a\theta^a)L + \theta^a\partial_a L = \partial_a(\theta^a L).$$

This gives

$$\Lambda^a = \theta^a L.$$

Use this in the Minkowski-metric version of the identity (20) to get[20]

$$\int_R \left(\Psi\,\delta\phi + \partial_a\left(\frac{\delta L}{\delta\,\partial_a\phi}\theta^b\partial_b\phi - \theta^a L\right) - T_H^{ab}\partial_a\theta_b\right) = 0 \tag{22}$$

with T_H^{ab} given by (10). This holds for all regions R, so it implies

$$\Psi\,\delta\phi + \partial_a(T_C^{ab}\theta_b) - T_H^{ab}\partial_a\theta_b = 0$$

with T_C^{ab} given by (7). Rearrange this to get the key result

$$\boxed{\Psi\,\delta\phi + (\partial_a T_C^{ab})\theta_b = (T_H^{ab} - T_C^{ab})\partial_a\theta_b.} \tag{23}$$

[19] Section 7.4 in Weinberg (1995) illustrates this for infinitesimal Lorentz transformations, and the next section shows an example.

[20] $\hat{L} = L$ after specializing to the Minkowski metric.

For translations, θ_b is constant ($\partial_a\theta_b = 0$), so (23) implies that $\partial_a T_C^{ab} = 0$ whenever the fields satisfy their equations of motion $\Psi = 0$. This reproduces the conservation law (8). The conservation law holds regardless of θ_b, so equation (23) reduces to

$$0 = (T_H^{ab} - T_C^{ab})\partial_a\theta_b.$$

whenever the fields satisfy their equations of motion ($\Psi = 0$). This holds for arbitrary non-constant θ_b, so it implies that T_C^{ab} and T_H^{ab} are equal to each other (for scalar fields) whenever the fields satisfy their equations of motion.

8 General proof for gauge fields

This section adapts the preceding derivation to models involving only abelian gauge fields (together with the background metric field) instead of scalar fields. A gauge field is represented here by the components A_a of a one-form field, which is a type of tensor field (article 09894). The key difference is that the effect of a fieldomorphism on a gauge field has an extra term compared to the scalar-field case (compare to equation (21)):

$$\delta A_a = \theta^b \partial_b A_a + A_b \partial_a \theta^b. \tag{24}$$

This is like the infinitesimal version of a coordinate transform (article 09894), but here regarded as a transformation of the field instead of as a transformation of the coordinate system.

Assuming that the action is invariant under the same set of fieldomorphisms as in the previous section leads to this analog of equation (22):

$$\int_R \left(\Psi^a \, \delta A_a + \partial_a \left(\frac{\delta L}{\delta \, \partial_a A_c} \big(\theta^b \partial_b A_c + A_b \partial_c \theta^b \big) - \theta^a L \right) - T_H^{ab} \partial_a \theta_b \right) = 0 \tag{25}$$

with

$$\Psi^a \equiv \frac{\delta L}{\delta A_a} - \partial_b \frac{\delta L}{\delta \partial_b A_a}$$

and with T_H^{ab} given by (10). This holds for all regions R, so it implies

$$\Psi^a \, \delta A_a + \partial_a \left(\frac{\delta L}{\delta \, \partial_a A_c} \big(\theta^b \partial_b A_c + A_b \partial_c \theta^b \big) - \theta^a L \right) - T_H^{ab} \partial_a \theta_b = 0. \tag{26}$$

If the the lagrangian depends on $\partial_a A_c$ only via F_{ac}, as usual (for gauge invariance), then the quantity

$$K^{ac} \equiv \frac{\delta L}{\delta \, \partial_a A_c}$$

is antisymmetric ($K^{ac} = -K^{ca}$), so the quantity

$$k^{ab} \equiv \partial_c \left(\frac{\delta L}{\delta \, \partial_a A_c} A^b \right)$$

is identically conserved ($\partial_a k^{ab} = 0$), as illustrated in section 5. Using this property of k^{ab} and the definition (7) of T_C^{ab}, equation (26) can be rearranged to get the key result

$$\boxed{\Psi^a\,\delta A_a + \partial_a T_C^{ab}\theta_b = (T_H^{ab} - T_C^{ab} - k^{ab})\partial_a\theta_b.} \tag{27}$$

Using the same logic as in the previous section, this one equation shows that T_C^{ab} is conserved *and* that it is equal to T_H^{ab} modulo an identically-conserved term whenever the field A_a satisfies its equation of motion $\Psi^a = 0$. The example shown in section 5 is a special case of this.

This approach generalizes immediately to models that include both scalar fields and gauge fields: equations (22) and (25) are subsumed into a similar equation that includes the $\delta L/\delta\cdots$ terms for both scalar fields and gauge fields. The conclusion is that T_C^{ab} and T_H^{ab} are equal to each other modulo an identically-conserved term whenever the fields satisfy their equations of motion, as illustrated in section 6.

9 References

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