Local Observables in Quantum Field Theory

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Abstract Quantum field theory (QFT) is the foundation for our current understanding of almost everything in nature except gravity. QFT builds on the general principles of quantum theory (article 03431) by adding the concept of a **local observable** – an observable associated with a bounded region of spacetime. This article introduces some of the general principles of QFT, expressed in terms of local observables.

Contents

1 The scope of this article

Historically, the subject(s) called **quantum field theory (QFT)** originated from the desire to construct nontrivial quantum models respecting the symmetries of flat spacetime, including Lorentz symmetry. Today, the name QFT is used with several different overlapping meanings, and any definition that tries to cover all of them is probably too generic to be useful. Instead of committing to a particular definition of QFT, this article focuses on one of its key concepts: the concept of a local observable.[1]

QFT does not require all observables to be local. A typical model includes some observables that are not localized in any bounded region of spacetime, and a subject called **topological QFT** considers models that don't have any local observables at all. Sometimes the mathematical structure of QFT is used in a different way, with observables that are localized in bounded regions of an *abstract* manifold that is not interpreted as spacetime.[2] All of these things are interesting and important, but this article doesn't try to cover them. This article focuses only on observables that are localized in bounded regions of spacetime. These are the most important observables in most practical applications of QFT.

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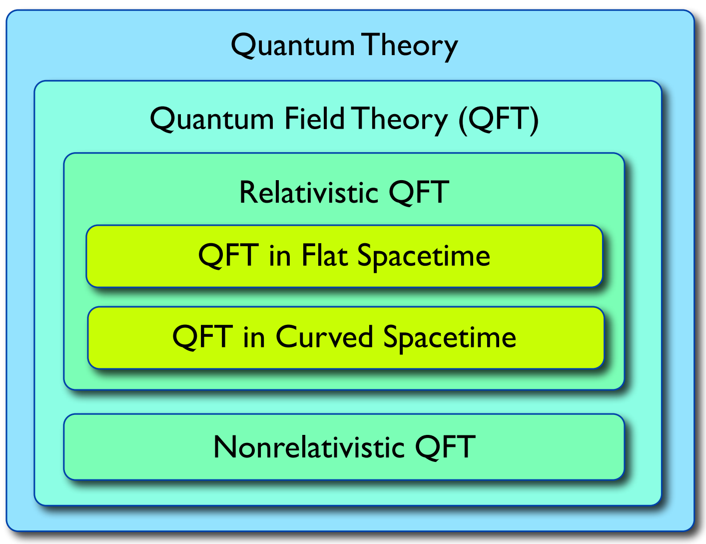
[1]In this article, an observable is called **local** if it is associated with a bounded region of spacetime, not necessarily a single point. Sometimes people use the term *local observable* to mean an observable associated with a single point, but that's not how I'm using the term here. Section 6.1.1 in Halvorson and Mueger (2006) explains why associating observables with individual points in a smooth spacetime can be problematic.

[2]A prominent example of this is the **AdS/CFT correspondence** (also called **gauge/gravity duality**), in which observables are localized on the "boundary" of an asymptotically anti-de Sitter spacetime (Polchinski (2010), Maldacena (2011), Sundrum (2011), Hubeny (2015), Kaplan (2016), Van Raamsdonk (2016)). That usage of QFT has given us a relatively good understanding of quantum gravity in spacetimes satisfying that asymptotic condition, but the real universe appears to be asymptotically de Sitter instead. That's why the first sentence in this article's abstract says "...except gravity." We do not yet have a good understanding of how to define quantum gravity in asymptotically de Sitter spacetimes.

2 Different usages of the name "QFT"

Physicists use the word *theory* to refer to anything from a general subject to an individual model. Similarly, the expression *quantum field theory* may refer to the general subject, or it may refer to an individual model.[3]

QFT is a refinement of quantum theory, in the sense that QFT adds to the list of conditions that any good model should satisfy. **Relativistic QFT** is a further refinement that adds a universal speed limit to the list of conditions. Sometimes *QFT* is used as a synonym for *relativistic QFT*, but **nonrelativistic QFT** is also an important subject.[4] Sometimes the name *relativistic QFT* refers specifically to **QFT in flat spacetime**, but **QFT in curved spacetime** is also an important subject.[5] This Venn diagram shows how the words are used in this article:



Most of the proposed axiomatic systems treat spacetime as a smooth manifold. That excludes **lattice QFT**, and the name *QFT* is often used with that exclusive connotation. In this article, *QFT* includes lattice QFT. Section 3 explains why.

[3]Example: "QED is a quantum field theory."
[4]Section 15
[5]Section 14

3 Emergent properties

Instead of studying individual models, we often study entire families of models by promoting some of their shared properties to axioms and then exploring the consequences of those axioms. People have tried to approach QFT this way, using various sets of axioms.[6] However, for many models of interest, some of the properties that we might like to promote to axioms are only approximately satisfied. The imperfections may be undetectably small within the model's intended range of applications, but still – things called *axioms* are meant to be exact.

As an example, consider quantum electrodynamics (QED). The only known nonperturbative[7] definitions of QED treat spacetime (or at least space) as a discrete array of points, typically a lattice. This way of defining a model is called **lattice QFT**. The discretization is artificial,[8] and that's okay, because QED isn't meant to be a Theory of Everything. We can take the lattice scale (the distance between neighboring points) to be much smaller than the finest scale resolved in any experiment to which QED is meant to apply. Then properties associated with continuous spacetime **emerge** at resolutions much coarser than the lattice scale.[9]

To accommodate important models like QED, this article distinguishes between properties that should hold at all scales, and properties that are allowed to be **emergent** – allowed to be violated near the lattice scale, as long as any violations are imperceptable at much coarser resolutions.[10] Causality principles (section 5) and spacetime symmetries (section 9) are allowed to be emergent.

[6]Examples include the **Wightman axioms** (Streater and Wightman (1980)), **algebraic QFT** (Haag (1996)), and what Schreiber (2008) calls **functorial QFT** (Monnier (2019)), although algebraic QFT can also be expressed as a functor (Brunetti and Fredenhagen (2004)). This article's approach is closest to algebraic QFT, which emphasizes local observables. For perspectives, see Tachikawa (2017a), Tachikawa (2017b), Freed and Seiberg (2021), Dedushenko (2022), Poland and Rastelli (2022).

[7]Most applications of QED rely on **perturbation theory**, the art of using small-parameter expansions, but everything makes more sense (both mathematically and physically) when we start with a *non*perturbative definition.

[8]The conventional concept of spacetime is expected to break down near the Planck scale, but in a more interesting way (Bousso (2002)), nothing like a simplistic discretization.

[9]Many models that differ from each other at high resolution become indistinguishable at sufficiently low resolution, a phenomenon called **universality** (McGreevy (2016) and article 10142).

[10]Witten (2017) contrast this with other usages of the word *emergence*.

4 Local observables

In quantum theory, the word **observable** is typically used for something that can be measured in a single measurement event.[11] As explained in article 03431, observables are represented by operators on a Hilbert space.[12,13] To specify a model, we need to specify its observables – at least we need to specify which elements of the $*$-algebra represent which measurable things.

One way to do this is by assigning a set $\Omega(R)$ of observables (represented by operators) to each region R of spacetime.[14] We call them **local observables** if R is bounded. Physically, observables in $\Omega(R)$ represent measurable things that are localized within that region.[15] Mathematically, all of the operators in all of the sets $\Omega(R)$ are represented by operators on the same Hilbert space $\mathcal{H}$.[13]

These sets should satisfy an obvious condition called **isotony**: if $R_1 \subset R_2$, then $\Omega(R_1) \subset \Omega(R_2)$. Any assignment that satisfies this condition is called a **net** of observables.

The direction of the association is important. The **time-slice principle** (article 22871) says that if R_3 is a neighborhood of one Cauchy surface[16] and R_4 is a neighborhood of another Cauchy surface, then $\Omega(R_3) = \Omega(R_4)$. That's why we assign a set $\Omega(R)$ of observables to each region R, instead of assigning a region to each (set of) observable(s).

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[11]The word *observable* is sometimes used more broadly, to include things like correlation functions whose measurement requires multiple measurement events. That's not how I'm using the word here.

[12]The word *observable* is used both for the measurable thing and for the operator that represents it.

[13] In QFT, to be more careful, we should say that observables are represented by elements of a $*$-algebra (article 74088) and that we can choose how that $*$-algebra is represented by operators on a Hilbert space (section 8).

[14]This article uses the Heisenberg picture (article 22871).

[15]When we associate observables with specific regions of spacetime, we are relying of the fact that the concept of an observable is primary in quantum theory: it does not depend on how the observable is measured. A measurement is a physical event. With some idealization, we could say that a measurement occurs within a particular region of space and within a particular interval of time, but that can only be an idealization because real measurements don't have well-defined boundaries. Even a quick measurement may be preceded by a long period of preparation and have side-effects that propagate over great distances for many years into the future.

[16] This article assumes that spacetime is **globally hyperbolic**, which roughly means that it admits a time coordinate t such that a hypersurface of constant t is a valid **Cauchy surface**. Section 3 in Witten (2019) introduces these concepts in detail.

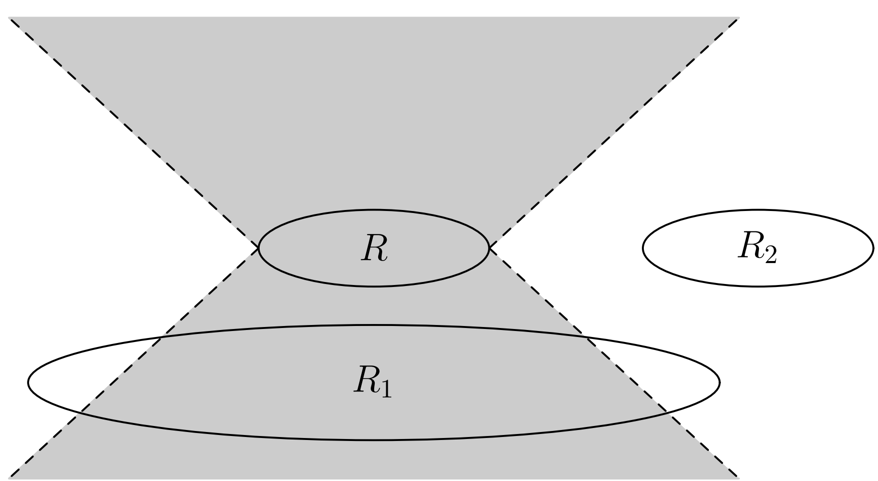
5 Causality properties

Relativistic QFT combines the idea of QFT with the idea of relativity – that influences cannot propagate faster than a limiting speed, the "speed of light." In relativistic QFT, the sets $\Omega(R)$ are supposed to have these **causality properties**, at least as emergent properties:

- If *every* timelike and lightlike worldline that intersects R also intersects R_1, then all of the observables in $\Omega(R)$ are included in the algebra generated by observables in $\Omega(R_1)$. This is a local version of the time-slice principle. It has a few different names (section 6).

- If R and R_2 are not causally connected (if no timelike or lightlike worldline intersects both of them), then all of the observables in $\Omega(R)$ commute with all of the observables in $\Omega(R_2)$. This principle has several different names, including **microcausality** (section 6).

These conditions don't assume that spacetime is flat. They apply in any globally hyperbolic spacetime,[16] whether flat or curved. The spacetime relationships in these conditions are illustrated below, where the shaded area represents the union of all timelike and lightlike worldlines through R:



6 Causality properties: some synonyms

The causality properties introduced in the previous section have various names. Names that have been used for the local version of the time-slice property include:

- a local version of the time-slice property,[17]
- the existence of a causal dynamical law,[18]
- local primitive causality.[19]

Names that have been used for what section 5 called *microcausality* include:

- microcausality,[20]
- Einstein causality,[21]
- causality,[22]
- locality,[23]
- causal locality.[24]

Beware that the words *causality* and *locality* are used with other meanings, too. The word *locality* is often used to describe the structure of the lagrangian in lagrangian formulations of QFT, and more generally for the gluing property in one prominent axiomatization of QFT.[25] The same word is also sometimes used as a synonym for *Bell locality*,[26] a fanciful property that does not hold either in quantum field theory or in the real world (article 70833).

[17] Brunetti and Fredenhagen (2004), definition 2.1
[18] Haag (1996) equations III.3.44 and III.1.10, and Verch (2001)
[19] Horuzhy (1990), section 1.2, axiom IVb
[20] Halvorson and Mueger (2006), section 2.1, assumption 2
[21] Halvorson and Mueger (2006), section 3. Don't confuse this with Einstein locality.
[22] Horuzhy (1990), section 1.2, axiom III; Haag (1996), equation III.1.4
[23] Horuzhy (1990), section 1.2, axiom III; Araki (1999), section 4.1, axiom 3; Summers (2008), section 2, page 4
[24] `https://ncatlab.org/nlab/show/causal+locality`
[25] Monnier (2015), section 1, page 3; Freed (2014), section 2.3
[26] Weihs *et al* (1998)

7 The impossibility of superluminal communication

Microcausality ensures that measurements performed in mutually spacelike regions cannot influence each other.[27] This section explains why.

Consider two regions of spacetime that are not causally connected. Let X be an observable localized in one of those regions, and let Y be an observable localized in the other one. Microcausality says that X and Y commute with each other. Suppose for simplicity that a measurement of X has only a countable list of possible outcomes, represented by projection operators $X_1, X_2, \ldots$, and similarly for Y. The fact that X and Y commute with each other implies[28,29]

$$\sum_n \rho(Y_m|X_n)\rho(X_n) = \rho(Y_m)$$

for any state $\rho(\cdots)$, so the distribution of Y-measurement outcomes is not affected by whether or not X is measured.

In general, we don't know how to predict the outcomes of individual measurement events. We can only predict the distribution of outcomes. We just proved that the distribution of outcomes in one region cannot be affected by what is being measured (or not) in the other region if those two regions are not causally connected, so events in one region cannot influence events in the other region in any way that we know how to predict. This is what "faster-than-light communication is impossible" means.

[27] Other conditions related to this are reviewed in Summers (2008), Fewster (2016), and section 3 in Halvorson and Mueger (2006).

[28] Articles 77228 and 03431 introduce this notation.

[29] Proof: The left-hand side may also be written $\sum_n \rho(X_n Y_m X_n)$. If X commutes with Y, then this equals $\sum_n \rho(Y_m X_n)$. Use $\sum_n X_n = 1$ to finish the proof.

8 Choosing a representation

Observables are normally represented by operators that operate on a Hilbert space, but they can also be represented as elements of an abstract noncommutative C*-algebra, which in turn can be represented as an algebra of operators on a Hilbert space. This perspective, using two layers of representation, can be enlightening when the same C*-algebra admits different Hilbert-space representations that are not unitarily equivalent to each other.

The general principles of QFT can be organized into two groups: those that refer only to the algebraic structure, and those that say something about which Hilbert-space representations are physically reasonable. The second group includes the requirement that the Hilbert space should be *separable* (articles 90771 and 03431). Other examples:

- The causality principles introduced in section 5 are in the first group: they refer only to the algebraic structure.
- The principle introduced in section 10 is in the second group: it is a condition on how the algebra should be represented on a Hilbert space.

This perspective is especially enlightening when considering QFT in curved spacetime. In a closed universe (a globally hyperbolic spacetime whose Cauchy surfaces are compact), one Hilbert-space representation is naturally distinguished from all the others. In contrast, in an open universe (whose Cauchy surfaces are not compact), no one representation is naturally distinguished, so we must choose one based on other criteria. This is reviewed by Witten (2021), section 2.4.

If we use a normalized positive linear functional to represent a state, as in article 03431, then we don't need an explicit Hilbert-space representation, but such a representation is still implied through the **GNS reconstruction theorem**,[30] so conditions on the Hilbert-space representation still imply conditions on the state.

[30] Fewster and Rejzner (2019), section 2.3, theorem 10

9 Symmetry

A unitary (or antiunitary) operator U on the Hilbert space is called a **symmetry** if[31]

$$U^{-1}\Omega(R)U = \Omega(R^U)$$

for all regions R, where the regions R^U are obtained from R by a smooth transformation of spacetime (a **diffeomorphism**).[32]

The study of relativistic QFT is dominated by QFT in flat spacetime, especially by models that have **Poincaré symmetry**, which means that the transformations $R \to R^U$ preserve the Minkowski metric. In lattice QFT, such spacetime symmetries cannot be exact, but they can be emergent (section 3).

In a model with time translation symmetry, the operator that generates translations in time is the observable corresponding to the system's total energy (article 22871). Similarly, in a model with symmetry under translations in a given spatial direction, the operator that generates those translations is the observable corresponding to that component of the system's total momentum. The generator of rotations in a given plane is likewise the observable corresponding to the system's total angular momentum in that plane.

Symmetries for which $R^U = R$ are called **internal symmetries**. They affect individual observables but do not mix the sets $\Omega(R)$ with each other.

This article focuses on observables. In QFT, observables are typically expressed in terms of auxiliary objects, like **field operators**, which are typically not observables by themselves. In that context, models often have symmetries that affect the auxiliary objects but that don't affect any observables.

[31]Harlow and Ooguri (2021) review a different (but related) way of defining symmetries in QFT.

[32]Article 93875

10 The spectrum condition in flat spacetime

The idea that energy can be transferred among different parts of the system is familiar.[33] However, we also know from experience that if we continue extracting energy from one part of the system, then eventually nothing is left in that location but empty space, from which no further energy can be extracted. This is why the concept of *empty space* makes sense.

In classical field theory, that kind of stability can be enforced by requiring the energy density – the component $T^{00}(x)$ of the stress-energy tensor (article 11475) – to be nonnegative everywhere. In relativistic QFT, this is not possible.[34] One thing we can do instead is to impose the **spectrum condition**, which roughly says that the *total* energy of a system has a finite lower bound. More precisely, it says that when the algebra generated by the model's observables is represented as an algebra of operators on a Hilbert space,[35] the quantity $\langle\psi|H|\psi\rangle$ should have a finite lower bound among all unit state-vectors $|\psi\rangle$ in the Hilbert space, where H is the hamiltonian, the generator of translations in time (article 22871). The spectrum condition can also be called the **positive-energy condition**, because we can add a constant term to the hamiltonian to shift the minimum energy to zero without affecting any of the model's predictions.[36] The Hilbert space may or may not have any state-vector that actually achieves the lower bound,[37] but if it does, such a state is called a **vacuum state** or a **ground state**.[38]

Some models admit more than one vacuum/ground state. The phenomenon called **spontaneous symmetry breaking** (SSB) is an example. If $|a\rangle$ and $|b\rangle$ are any two states having the lowest possible energy, then any superposition (linear

[33]This idea can be made more precise using the **stress-energy tensor** (section 14).

[34]Section 2 in Fewster (2005a) shows a concise and general proof.

[35]Fewster and Rejzner (2019) and Witten (2021) explain the reason for this qualification.

[36]This is true because we are only considering models in which the spacetime metric is a prescribed *background field* (section 14), which is not influenced by the model's quantum entities.

[37]Witten (2021)

[38]The name *vacuum state* is more common in relativistic quantum field theory, where the lowest-energy state represents completely empty space. The name *ground state* is more common in nonrelativistic models of condensed matter, where some minimal amount of matter is built into the model, even in the lowest-energy state.

combination) of $|a\rangle$ and $|b\rangle$ also has that same energy. The names *vacuum/ground state* are usually reserved for a special subset of these lowest-energy states, namely those that cannot be mixed with each other by any observables that we could feasibly measure in practice. As explained in article 03431, this condition selects a special set of mutually orthogonal states and excludes their superpositions. These special states can also be recognized as those that have the **cluster property**:[39] the correlation between two localized clusters (products) of observables decreases as the distance between the clusters increases.

The spectrum condition is expressed as a condition on the *total* energy, but in relativistic QFT, it may imply a local stability condition like the one described at the beginning of this section.[40] Examples suggest that when the spectrum condition is satisfied, the energy density integrated[41] over any open region of spacetime can be negative, but not arbitrarily negative.[42] Ford (1978) highlights the importance of this kind of stability for thermodynamics. Section 12 highlights another application: it provides a local version of the concept of empty space,[43] which in turn is the foundation for the *particle* concept in not-necessarily-flat spacetime.

The spectrum condition also has other important implications in relativistic QFT. It is one of the inputs to the **spin-statistics theorem** (the origin of the Pauli exclusion principle)[44] and to the **CPT theorem** (which relates to the existence/definition of antiparticles).[45]

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[39] Weinberg (1996), section 19.1, pages 165-167; Haag (1996), equation III.3.19; Araki (1999), section 4.3, page 87; Streater and Wightman (1980), equation 3-37

[40] Fewster (2005b)

[41] This means the T^{00} component of the stress-energy tensor integrated with a suitable weighting function.

[42] This has been shown for models of free fields, for models with conformal symmetry in two-dimensional spacetime, and for one two-dimensional model without conformal symmetry. Reviews include Fewster (2012) and Cadamuro (2019). Ford *et al* (2002) showed that the region must be *open* in the topological sense: if the energy density were integrated only over a region of space, with no extent in time, then it would not have a lower bound.

[43] By the way, stability is one of the main reasons for studying **supersymmetry**. In nonperturbative formulations of quantum gravity (article 03431 cites some references), supersymmetry seems to be important for the emergence/stability of spacetime itself. This is mentioned at the end of section 2.1 in Banks (1998), below equation (84) in Bigatti and Susskind (1997), and at the end of section 2 in Banks (2013).

[44] Streater and Wightman (1980), Greenberg (1998), and section II.5.1 in Haag (1996)

[45] Witten (2015), section 5.8 in Weinberg (1995), and the references in footnote 44

11 The Reeh-Schlieder property

Relativistic models have a property called the **Reeh-Schlieder property**, which roughly says that for any open region R in spacetime, no matter how small,[46] and for almost any single state-vector $|\psi\rangle$, all other state-vectors can be generated from $|\psi\rangle$ using only observables in $\Omega(R)$.[47] The only requirement on $|\psi\rangle$ is that it has bounded total energy-momentum in the sense defined in Witten (2018), section 2.3. A vacuum state is one example of such a state, so the Reeh-Schlieder property implies that all states can be generated from a vacuum state using observables in $\Omega(R)$.

Thanks to microcausality, this implies[48] that $|\psi\rangle$ cannot be annihilated by any observable localized in any bounded region of spacetime. For any measurement of such an observable, all of its possible outcomes have a nonzero (possibly very small) probability of occurring. That's because each possible outcome is represented by a projection operator P localized in that region, and $P|\psi\rangle \neq 0$ (implied by the Reeh-Schlieder property) combined with $P = P^*P$ (true for any projection operator) implies $\langle\psi|P|\psi\rangle \neq 0$.[49]

In flat spacetime, the Reeh-Schlieder property is a theorem that follows from other relatively mild assumptions.[50] A curved spacetime looks approximately flat within a sufficiently small region, so something like the Reeh-Schlieder property should also hold for relativistic models in curved spacetime.[51] The Reeh-Schlieder property does not necessarily apply in nonrelativistic models.

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[46]Saying that R is *open* implies that it has finite extent in every dimension, so R can't be a single point.

[47]More carefully: starting with $|\psi\rangle$, every state-vector can be arbitrarily well-approximated by applying operators in the von Neumann algebra generated by $\Omega(R)$.

[48]Witten (2018), section 2.4

[49]If P is any projection operator in $\Omega(R)$, then the state $P|\psi\rangle$ is clearly annihilated by an operator in $\Omega(R)$, namely by $1 - P$. This doesn't contradict the Reeh-Schlieder property. It just means that the state $P|\psi\rangle$ cannot have strictly bounded total energy-momentum, even though it may be bounded for all practical purposes.

[50]Section 5.2 in Fewster and Rejzner (2020), section 2 in Witten (2018), section 2.3 in Halvorson and Mueger (2006), theorem 4.14 in Araki (1999), section II.5.3 in Haag (1996), and theorem 1.3.2 in Baumgärtel (1995).

[51]Sanders (2009), Dappiaggi (2011)

12 Particles

In QFT, observables are tied to spacetime, not to particles. A particle is a phemomenon, often a transient phenomenon. It is something that a model predicts, not part of a model's axioms.[52] Particles are described using observables that detect their presence in specified locations, just like how particle detectors work in the real world. This will be illustrated in a separate article, using a simple example of a relativistic QFT.

Stable particle species in flat spacetime are often characterized in terms of irreducible representations of the Poincaré group, the group of symmetries of flat spacetime.[53] Given one state that corresponds to a single particle, all Poincaré transforms of that state should represent the same kind of particle. Conversely, given two single-particle states corresponding to two different particle species, those states cannot be mixed with each other by Poincaré transforms. This way of characterizing particle species is convenient when it works, but it doesn't work in a generic curved spacetime that doesn't have any symmetries, and it doesn't work for unstable particles.[54] Using observables representing localized particle-detectors to characterize particles is not as pristine as using Poincaré symmetry, but it is more flexible and is closer to the way particles are actually characterized in a laboratory.

Ideally, an operator representing a particle detector should have zero probability of detecting anything in a vacuum state, because a vacuum state is supposed to represent completely empty space. In relativistic QFT, that idealization can be realized to an excellent approximation, but it can't be realized exactly, for a few reasons. First, the Reeh-Schlieder theorem (section 11) says that an observable that is strictly localized in a bounded region of spacetime cannot annihilate

[52]Weinberg (1995) explains how to systematically construct a model that has a given particle content. That doesn't contradict the point I'm making here. Constructing a model so that it exhibits a desired phenomenon doesn't necessarily mean that we should think of that phenomenon as one of the theory's axioms.

[53]Weinberg (1995), section 2.5

[54]It doesn't work for unstable particles because even if $|\psi\rangle$ is a state corresponding to a single unstable particle at a given time, the same state (in the Heisenberg picture) or its time-translations (in the Schrödinger picture) will involve more than one particle after the unstable particle decays (or before it forms), so as far as the Poincaré-symmetry approach is concerned, $|\psi\rangle$ isn't really a single-particle state!

the vacuum state, so an operator representing a particle-detector must either be imperfectly localized (so that it can annihilate the vacuum state), or it must be noisy (have a small probability of registering something even in the vacuum state). Second, we can consider observables representing detectors that are accelerating,[55] In QFT, detectors undergoing extreme acceleration have a significant probability of detecting something even in a state where non-accelerating detectors do not. This is called the **Unruh effect**. Third, the usual definition of the vacuum state itself relies on special symmetries of flat spacetime (section 10). A generic curved spacetime has those symmetries only approximately, within sufficiently small regions. We can still use a localized version of energy (the integral of T^{00} over a bounded region) to define a local concept of vacuum, and we can still use the localized detector concept to define *particle*, but then a generic time-dependent metric can lead to spontaneous particle production.[56] **Hawking radiation** from black holes is a famous example of this phenomenon.[57]

This is the key message:

> Particles are phenomena, not axioms.

Like most phenomena, *particle* does not have any definition that is perfectly natural and unambiguous. That's okay. Quantum field theory can (presumably)[58] describe phenomena like rivers, too, even though *river* also doesn't have any definition that is perfectly natural and unambiguous. The lack of a perfect definition may be a nuisance when trying to prove theorems, but it is not a fundamental problem.

[55] I'm referring to acceleration in the *absolute* sense. An object resting on the surface of the earth is accelerating in the absolute sense (it has a nonzero weight). An object in orbit is not accelerating in the absolute sense (it is weightless).

[56] Su (2017), section 1.8

[57] This example is famous because it implies that black holes eventually evaporate, which in turn leads to important clues (in the form of paradoxes) about how gravity and quantum physics fit together.

[58] `https://physics.stackexchange.com/a/702712`

13 Nuclearity conditions

This is a topic that introductions to QFT rarely mention, even though it is part of the foundation for statistical mechanics (article 23206). Precise mathematical formulations of the idea are relatively technical, but the basic idea is simple:[59] the number of mutually orthogonal states available to a system with limited energy and volume should be finite. One way to make this idea mathematically precise is through a **nuclearity condition**, like the one proposed in Buchholz and Wichmann (1986).[60] According to page 326 in that paper, a condition like this is expected to be satisfied "by any local quantum field theory admitting a particle interpretation and having regular thermodynamical properties."

Section V.5 in Haag (1996) reviews various formulations of this condition and its relationship to local observables.

[59]Buchholz and Porrmann (1990), pages 237-238: "based on the heuristic idea that the number of [orthogonal] states of fixed total energy and limited spatial extension should be finite. ... the mathematical description of this idea is somewhat subtle and has led to different formulations."

[60]This is reviewed concisely in section 4 of Fewster (2005b), and Fewster *et al* (2005) offers some clarifications.

14 Background fields

In classical physics, as an approximation, we often consider models with **background fields** – things that influence, but are not influenced by, the model's dynamic entities. Background fields may be functions of space and time, but their (space- and time-dependent) configuration is a prescribed input to the model, along with other fixed inputs to the model like mass parameters and coupling constants. The background-field approximation is often sufficient, and it tends to be easier than treating those fields as dynamic entities.

The same idea can be used in QFT. The electromagnetic field is a prominent example. In quantum electrodynamics, the electromagnetic field is a quantum field that both influences and is influenced by the electron/positron field (and any other charged matter fields we might want to include), but for some purposes we can use a much easier model in which the electromagnetic field is a prescribed classical background field instead.

In quantum theory, the spacetime metric – general relativity's representation of the "gravitational field" – is almost always treated as a classical background field,[61] because (nonperturbative) quantum gravity is a much more challenging subject that is not yet well-understood.[62,63] Nonperturvatively, treating the spacetime metric as an independent quantum field is probably not even the right thing to do.[64] For most practical applications, treating the gravitational field (the metric field) as a classical background field is sufficient, and we can usually even take it to be flat (not curved), as in most of the literature about QFT. Most of the general principles of relativistic QFT, including the causality principles introduced in section 5, assume a prescribed metric field.

[61] Witten (2021), Fewster and Verch (2015), Wald (2009), Parker and Toms (2009), Birrell and Davies (1982)

[62] In contrast to nonperturbative quantum gravity, treating a *small perturbation* of the spacetime metric as a quantum field is relatively straightforward (Donoghue (1995)). This is usually sufficient because gravity is so weak.

[63] Some experiments (like Colella *et al* (1975)) have explored the effect of "gravity" on quantum systems, but they are really only exploring the effect of absolute acceleration in flat spacetime.

[64] The holographic principle (reviewed in Bousso (2002)) is a strong clue that a different approach is required, as emphasized in Jacobson (1995) and as demonstrated by nonperturbative formulations string theory (Lashkari *et al* (2014) and references cited in article 03431).

15 Not-necessarily-relativistic QFT

In applications where everything is moving very slowly compared to the limiting speed, the existence of a limiting speed doesn't matter. For such applications, we can make the math easier by using a model that does not have any limiting speed. **Nonrelativistic QFT** is QFT without a limiting speed.

To accommodate not-necessarily-relativistic models, the causality conditions that were introduced in section 5 can be replaced by less-restrictive conditions. The local time-slice property is replaced by the basic time-slice property (article 22871), and microcausality is replaced by what can be called **equal-time commutativity**: observables associated with different places at the same time should commute with each other. These conditions assume that a preferred time coordinate has been chosen, as usual in nonrelativistic models.[65] These less-restrictive conditions allow for the existence of a limiting speed, but they don't require it.

Some models, like nonrelativistic quantum electrodynamics (NRQED), are neither strictly relativistic nor strictly nonrelativistic: the nonrelativistic approximation is built into the matter fields, but the quantum electromagnetic field prevents the model from being strictly nonrelativistic. It can be reduced to a strictly nonrelativistic model by omitting the quantum electromagnetic field and keeping only the matter fields with a static Coulomb interaction. This is good enough for much of **quantum chemistry** (applications of quantum physics to chemistry).

[65] In relativistic QFT, these conditions hold with respect to *any* time coordinate – any coordinate t with the property that every hypersurface of constant t is a spacelike Cauchy surface.

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