The Core Principles of Quantum Theory and the Nature of Measurement

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Abstract Quantum theory is the foundation for our current understanding of most natural phenomena. The core principles of quantum theory, namely Born's rule and the state-update rule, revolve around measurement. The distinction between *measured* and *not measured* is unavoidably fuzzy in principle (even if it is clear enough in practice), but using quantum theory requires drawing that line somewhere. That situation is called the **measurement problem**. This article introduces the core principles of quantum theory with emphasis on their relationship to the physical process of measurement.

To help clarify the content of the core principles, an observable is represented by a set of projection operators representing the possible outcomes when the observable is measured, and states (which are used to tell quantum theory what we know so far about the system's history) are represented by positive linear functionals on the algebra of observables.

Contents

1 Introduction

The models currently used in physics can be organized into two categories, differing from each other in the way they treat **observables** (things that can be measured in a single measurement event):[1]

- In **classical models**, all observables are assumed to be compatible with each other, so we can treat all of the model's observables as having well-defined values whether or not they are being measured.
- In **quantum models**, many observables are *not* compatible with each other, and they can't all have well-defined values when they're not being measured.[2]

The way classical models treat observables might seem consistent with everyday experience, but everyday experience is only a small subset of everything we want to understand about nature. The most accurate and comprehensive models we have today are based on the general principles of quantum theory.[3]

The statement that two *observables* are not compatible with each other is stronger than the statement that two *measurement processes* are not compatible with each other. Measurement is a physical process, and clearly some physical processes are not compatible with each other even if they are individually allowed. Still, any given observable can be measured in many different ways, so we might expect that we could always find mutually compatible ways of measuring any given set of observables. In quantum theory, observables themselves can be incompatible with each other, so *every* way of measuring one of them is incompatible with *every* way of measuring the other one, unless we compromise the quality of the measurements (section 22).

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[1] The word *observable* is often used more broadly, to include things like correlation functions whose measurement requires multiple measurement events, but that's not how I'm using the word here. The thing that I am calling a *measurement event* corresponds to what Peres (2002) calls a **test** and to what Kraus (1983) and Ludwig (1983) call an **effect**. It is usually just called a *measurement*.

[2] This is the significance of the **Kochen-Specker theorem** (article 77228).

[3] Quantum theory includes quantum field theory as a special case (article 21916).

2 Observables

To represent observables mathematically, we can start with a **Hilbert space** $\mathcal{H}$ (article 90771). A property that the system could have is represented by a projection operator[4] P on the Hilbert space $\mathcal{H}$. The complementary projection operator $1 - P$ represents the opposite property. The pair $\{P, 1 - P\}$ is an example of an observable. The projection operators P and $1 - P$ represent the possible outcomes when this observable is (perfectly) measured. More generally, a list of projection operators

$$\{P_1, P_2, P_3, ...\} \tag{1}$$

satisfying

$$\sum_n P_n = 1 \tag{2}$$

can be used to represent an observable. The projection operators P_n represent the possible outcomes when this observable is (perfectly) measured. Article 74088 shows that the condition (2) implies

$$P_j P_k = 0 \text{ if } j \neq k. \tag{3}$$

In words, these projection operators are **orthogonal** to each other.[5]

Different formulations of quantum theory handle time in different ways (article 22871). This article uses the **Heisenberg picture**, in which all time-dependence is carried by the observables. In the Heisenberg picture, the "same" observable at different times is represented by different operators.[6]

[4]Recall that a **projection operator** P is an operator that equals its own adjoint and equals its own square. In this article, **operator** always means a linear operator on the Hilbert space – but see footnote 11 in section 5.

[5]Quantum theory uses a *separable* Hilbert space, in which any set of mutually orthogonal nonzero projection operators must be countable (Debnath and Mikusiński (2005), theorem 3.4.25). Observables with a continuum of possible outcomes can also be defined, but they can only be imperfectly measured.

[6]In quantum field theory (article 21916), the "same" observable in different *places* is likewise represented by different operators. The Heisenberg picture treats space and time similarly, so it is natural in relativistic models.

3 Packaging an observable as a single operator

An observable with a countable list of possible outcomes can be represented by a list of projection operators P_n that sum to 1, as in the previous section. Each of those projection operators represents one of the possible outcomes when that observable is measured. For convenience, we can represent an observable as a single self-adjoint operator instead:

$$A = a_1 P_1 + a_2 P_2 + a_3 P_3 + \cdots , \tag{4}$$

where the coefficients a_n are distinct real numbers. Article 74088 shows that the projection operators P_n can be recovered from the single operator A, so we don't lose any information by using the single operator A to represent the observable. We can think of the coefficients a_n as labels for the different possible outcomes. Different choices of the labels a_n give different representations of the same observable, because a measurement still has the same list of possible outcomes, represented by the same projection operators P_n. Only the labels are different.[7]

Assigning distinct numeric labels (a_n) to the different possible outcomes (P_n) can be convenient for at least two reasons. First, this enables using numeric measures like the mean and variance when describing the distribution of measurement outcomes. Second, sometimes the same operator also plays another important role in the model, a role for which the values of the coefficients do matter. In particular, operators that generate symmetries are often also used as observables. Examples include the operators that generate translations in time, translations in space, and rotations, which are (by definition) the operators that represent energy, momentum, and angular momentum, respectively. In such cases, thinking of those same special coefficients as part of the definition of the *observable* is natural, even though the coefficients are not needed for the general principles introduced in this article.

[7]Some textbooks assert that the outcome of a measurement *is* one of the coefficients a_n. They really just mean that the measurement device can be designed so that it displays the number a_n when the outcome P_n is obtained. Given any such device, we can modify it to display a different number a'_n when the outcome P_n is obtained, but it's clearly still measuring the same physical observable.

4 States

To specify a model, we specify its observables. To apply a model, we need to tell quantum theory what we know about the system's physical state – what we know about how the physical system was prepared.[8] We can do this by specifying a **normalized positive linear functional**, also called a **state** because of how it's used in quantum theory. Using the word *state* as a synonym for *normalized positive linear functional* is common in the math literature. *State* and *observable* are two examples of words that are used both for a purely mathematical entity and for the non-mathematical thing that it's used to represent.

Mathematically, a state ρ is a function that can take any operator A as input and returns a complex number $\rho(A)$ as output, subject to the conditions shown in article 77228. Those conditions imply

$$0 \leq \rho(P) \leq 1 \tag{5}$$

for any projection operator P.

For any state $\rho(\cdots)$ and any operator M such that $\rho(M^*M) \neq 0$, we can construct another state like this:[9]

$$\rho(\cdots|M) \equiv \frac{\rho(M^* \cdots M)}{\rho(M^*M)}, \tag{6}$$

where M^* denotes the adjoint[10] of M. The construct (6) plays an important role in the following sections.

[8] For readers who know what ψ-ontic and ψ-epistemic mean: I'm not committing to either of those types of interpretation here. When I say that a state represents what we know, I am simply describing how quantum theory is actually used, without presuming that it is or isn't "complete."

[9] The notation $\rho(\cdots|M)$ isn't standard, but it's useful. It is deliberately similar to the standard notation for a conditional probability, which is related to how the construct (6) is used in quantum theory.

[10] This notation for the adjoint of M is common in the mathematics literature. In the physics literature, it is more commonly denoted $M^\dagger$.

5 Using a Hilbert space to construct states

The general concept of a state (a normalized positive linear functional) does not rely on a Hilbert space.[11] In practice, though, a state is usually expressed like this:

$$\rho(A) = \frac{\sum_n \langle n|A|n\rangle}{\sum_n \langle n|n\rangle} \tag{7}$$

for all operators A, where $|1\rangle, |2\rangle, ...$ is some list of nonzero vectors in the Hilbert space on which the operators act. This can also be written $\rho(A) = \text{trace}(A\mu)$, where[12]

$$\mu \equiv \frac{\sum_n |n\rangle \langle n|}{\sum_n \langle n|n\rangle}$$

is called the **density matrix**.[13] The vectors $|n\rangle$ don't need to be orthogonal to each other, and they don't need to be unit vectors.

The simplest case is

$$\rho(A) \equiv \frac{\langle\psi|A|\psi\rangle}{\langle\psi|\psi\rangle}, \tag{8}$$

using just one vector. In this case, the vector $|\psi\rangle$ is called the **state-vector**. If the original state has the form (8), then replacing

$$\rho(\cdots) \to \rho(\cdots|M)$$

is equivalent to replacing

$$|\psi\rangle \to M|\psi\rangle.$$

[11] Witten (2022) illustrates the value of this generality.

[12] The notation in the numerator is common in the physics literature, but this article doesn't use it, so I won't bother defining it.

[13] The density matrix is often denoted ρ, which should not be confused with the (closely related) way I'm using the symbol ρ in this article.

6 The nature of quantum theory's predictions

When two observables are measured sequentially, the outcome of the second measurement typically cannot be predicted with certainty, no matter how carefully we try to prepare the system the same way every time. Sometimes we can attribute this apparent unpredictability to microscopic details that we know are beyond our control, but the observed violations of **Bell inequalities** (article 70833) teach us that some unpredictability cannot be explained in conventional ways.

Instead of trying to predict individual outcomes, quantum theory assigns probabilities to each of the possible outcomes.[14,15] The next section introduces the rules governing these assignments.

Quantum theory is testable even though it "only" assigns probabilities, because assigning probabilities to the outcomes of individual trials is effectively equivalent to predicting the distribution of outcomes in a large number of trials. Here's an example to illustrate the equivalence. Suppose we repeat an experiment a million times (a million **trials**). The outcome of each trial can be either X or Y. Even if we cannot predict which one of these will occur in a given trial, we might still be able to say something about the relative proportions of X and Y outcomes in a million trials. If quantum theory assigns probabilities 0.3 and 0.7 to the outcomes X and Y, respectively, then it is effectively assigning a probability very close to 1 to the collective possibility that the proportions of outcomes X and Y will be very close to 30% and 70% in a million trials. Assigning a probability very close to 1 is equivalent to making a prediction. In this practical sense, saying that quantum theory assigns probabilities to individual outcomes is equivalent to saying that it predicts the distribution of outcomes over a sufficiently large number of trials.

[14] I'm using the word **probability** in the Bayesian sense. For an unpolished but still excellent introduction to probability theory, see Jaynes (2003). It's unpolished because the author died before it was finished.

[15] In some cases, like when measuring the location of a macroscopic object, quantum theory *does* predict the individual outcome, because the probability assigned to one (very narrow range of) outcome(s) is very close to 1. Quantum theory is consistent with the reproducible aspects of everyday experience.

7 Born's rule and the state-update rule

At its core, quantum theory has two rules. Expressed in the Heisenberg picture, the rules are:

- **Born's rule.** If we start with a state ρ, and if P represents one of the possible outcomes of the next measurement, then $\rho(P)$ is the probability that quantum theory assigns to this outcome. This is consistent with (5).
- **The state-update rule.** If a measurement has occurred, and we know that the outcome is P, then we should replace the original state $\rho(\cdots)$ with the new state $\rho(\cdots|P)$, as defined in (6).[16] This new state is then used to apply Born's rule to the next measurement.

The state-update rule assumes that the measurement is perfect – that is, it perfectly resolves all of the possible outcomes from each other. Section 14 explains how to handle imperfect measurements.

Here are some easy consistency checks:

- The condition (2) implies $\sum_n \rho(P_n) = 1$, so exactly one of the outcomes in (1) will occur.
- Equation (3) implies $\rho(P_j|P_k) = 0$ if $j \neq k$. Interpreting P_j and P_k as mutually exclusive outcomes is consistent with this.

To motivate the notation (6), notice what happens when we iterate these rules. If P represents the outcome of the measurement that was just completed, and if another projection operator Q represents one of the possible outcomes of the next measurement, then the rules shown above say that the probability of this outcome is $\rho(Q|P)$. This is deliberately similar to the standard notation for a conditional probability. Article 77228 shows that the condition $\rho(P) = 1$ is equivalent to $\rho(\cdots) = \rho(\cdots|P)$, so saying that the outcome P is certain to occur is the same as saying that the state is already conditioned on that outcome.

[16] We don't need to worry about the fact that the denominator of $\rho(\cdots|P)$ is zero when $\rho(P) = 0$, because Born's rule says that the outcome P doesn't occur unless $\rho(P) > 0$.

8 From general principles to specific models

The principles introduced in in section 7 are very general. To apply them, we need to specify a **model**: we need to specify what the physical observables are and which operators are used to represent them.[17] Many different models are used, ranging from nonrelativistic single-particle quantum mechanics to relativistic quantum field theory.[18] Most models have several approximations or other limitations built into them. That makes the math easier. Some models, such as the Standard Model of particle physics, encompass most of what we currently know about nature. The principles in section 7 apply to all of these models. Examples of specific models will be introduced in separate articles.

[17] I'm using the word *model* the way it's used in names like "the Standard *Model* of particle physics." The word *theory* is often used the same way. Sometimes the word *model* may refer also to a specific state in addition to the collection of observables.

[18] Sometimes the name *quantum mechanics* is used a synonym for *quantum theory*, but sometimes *quantum mechanics* is used with a more specific connotation, similar to the usual distinction between *classical mechanics* and *classical field theory*.

9 Incompatible observables

This section illustrates the fact that in quantum theory, some observables are not compatible with each other: they cannot be measured simultaneously.

Suppose that all observables are represented as operators on a given Hilbert space $\mathcal{H}$. Consider a state of the form (8) and two observables of the form

$$A = a_1 P_1 + a_2 P_2 + a_3 P_3 + \cdots$$
$$B = b_1 Q_1 + b_2 Q_2 + b_3 Q_3 + \cdots ,$$

where the P_ks are mutually orthogonal projection operators, and so are the Q_ks. Suppose that $(AB - BA)|\psi\rangle \neq 0$ for all nonzero vectors $|\psi\rangle \in \mathcal{H}$. According to the state-update rule, if both observables are measured, with outcomes P_j and Q_k, respectively, then the original state-vector $|\psi\rangle$ should be replaced with either $P_j Q_k|\psi\rangle$ or $Q_k P_j|\psi\rangle$, depending on which observable is measured first. If they're measured simultaneously, then the result should be the same either way. That would imply $(AB - BA)|\phi\rangle = 0$ with $|\phi\rangle \equiv P_j Q_k|\psi\rangle = Q_k P_j|\psi\rangle$, which contradicts our assumption that $AB - BA$ doesn't annihilate any vectors. This indicates[19] that two such observables cannot be measured simultaneously.

[19] This argument only considers states of the form (8) in one Hilbert-space representation, even though a given operator algebra may admit many inequivalent Hilbert-space representations.

10 Choosing the initial state

To use Born's rule, we need to specify a state (section 4). How do we decide which mathematical state we should use to represent what we know about how the physical system was prepared? The answer is to use the state-update rule.

Here's how it works. Make a list of properties that we know the system had. We can represent this knowledge using a list of projection operators, because any potentially-observable property is represented by a projection operator. Starting with the *arbitrary* state $\chi(\cdots)$, apply the state-update rule for each one of the projection operators in this list, in chronological order,[20] just like we would if those properties were known from the outcomes of a sequence of deliberate measurements. If the outcome of the first "measurement" was P, then the state-update rule tells us to replace $\chi(\cdots)$ with $\chi'(\cdots) \equiv \chi(\cdots|P)$. If the outcome of the second "measurement" was Q, then the state-update rule tells us to replace $\chi'(\cdots)$ with

$$\chi''(\cdots) \equiv \chi'(\cdots|Q) = \chi(\cdots|QP).$$

And so on. We just keep iterating the state-update rule until we've accounted for everything we already know that might be relevant for making predictions about subsequent measurements of interest. The result will be a state of the form[21]

$$\rho(\cdots) = \chi(\cdots|A), \tag{9}$$

where A is the relevant product of projection operators:

$$A \equiv \cdots QP. \tag{10}$$

The state (9) the *initial* state we use when applying Born's rule to the measurement of interest. If the list of projection operators in A is complete enough – that is, if we know enough about how the system was prepared – then the probabilities assigned by Born's rule will depend only on the product (10) of projection operators, not on the arbitrary state χ, at least for the measurement of interest.

[20] The condition "in chronological order" is important because the operators don't necessarily commute with each other.

[21] This state is well-defined as long as the denominator $\chi(A^*A)$ is not zero. This is the only restriction on χ.

11 Choosing the initial state, more generally

Since quantum theory's predictions are probabilistic anyway, we might as well generalize the preceding construction to cases where we're not sure exactly how the physical system was prepared. Instead of a single product of projection operators as in (10), we can consider several different products A_n of projection operators, each of which represents a possible preparation. If $p(n)$ denotes the probability we assign to the nth possibility,[22] with $\sum_n p(n) = 1$, then we can take the state to be

$$\rho(\cdots) = \sum_n \chi(\cdots|A_n)p(n) \tag{11}$$

for arbitrary χ.

To check that the formula (11) is consistent with the principles introduced in section 7, suppose that the A_ns differ from each other only in the outcome of the last measurement in the sequence, so $A_n = P_n A$ for some list of projection operators P_n with $\sum_n P_n = 1$. Suppose that we don't know the outcome of that last measurement because it hasn't occurred yet. If we use the state (11) to represent this situation, then the probabilities that Born's rule assigns to the possible outcomes of the last measurement – the one that hasn't occurred yet – are

$$\rho(P_j) = \sum_k \chi(P_j|A_k)p(k) = \sum_k \chi(P_j|P_k A)p(k) = p(j),$$

because the orthogonality of the P_ns implies $\chi(P_j|P_k A) = \delta_{jk}$ for any χ and any A. Altogether, using the state (11) to represent this situation ensures that the probabilities $\rho(P_j)$ assigned by Born's rule will be consistent with the probabilities $p(j)$ that we already assigned.

[22] I'm using the letter p for these probabilities instead of the Greek letter ρ, because these probabilities don't necessarily come from Born's rule. We can't use Born's rule until *after* we've chosen the initial state.

12 Practical irreversibility in closed systems

In addition to specifying a model (section 8), we also need to specify a state (sections 10-11). Even if we're using a relatively comprehensive model, like one that is capable of describing something as complex as an oak tree with a subatomic level of detail, we might still choose a state that represents something relatively simple, like one or two electrons. Much of the subject called *particle physics* focuses on relatively simple states, even though the model may be relatively comprehensive.

In contrast, most familiar phenomena involve situations that are very complex, at least on a microscopic scale. Such complexity can lead to phenomena that are practically irreversible, even if the laws that govern them are fundamentally reversible. This is important for describing measurement as a physical process (sections (14)-(18)), at least if we're modeling a **closed system**, one that neither influences nor is influenced by anything outside of itself. The physical process of measurement can be described by using a state that includes the microscopic details of the measuring equipment – probes, recording devices, and so on – as part of the overall closed system. The complexity of the microscopic details can lead to phenomena that are practically irreversible, as required by the definition of *measurement* (section 14).

Much of this article is written with complex states in mind, because one of the central themes in this article is how the principles introduced in section 7 relate to the physical process of measurement. The principles in section 7 don't require treating measurement as a physical process within the model itself, but doing so can hep us understand why quantum theory works so well.

13 The measurement problem

The principles introduced in section 7 are the foundation for the most accurate and comprehensive models of nature that we have ever had, so the fact that they refer only to measurement outcomes might seem strange. Shouldn't any good theory describe what happens between measurements, too? One way to answer that question is: why should it? The only predictions that matter are the ones we can test, and we can only test the things we can observe (measure). Whatever a theory might say about what happens between measurements can only be tested indirectly, through whatever effects it has on eventual measurements.

In quantum theory, we can't pretend that observables have perfectly well-defined values if they're not actually measured, because observables are not all compatible with each other (section 9). This leads to an ambiguity called the **measurement problem**: to use quantum theory correctly, we need to know which observables are actually measured, but a completely natural and unambiguous definition of *measured* cannot exist! Measurement is a physical process, and distinguishing it from other physical process requires making arbitrary choices – like the arbitrary choices we make to define the difference between *rivers* and *streams*.

The measurement problem is not specific to quantum theory. The root of the problem comes from the fact that measurement is itself a physical process and is also our only means of testing predictions about physical processes. Peres (2002) said it this way:[23]

> In every physical situation *something* must remain unanalyzed. This is not a flaw of quantum theory, but a logical necessity in a theory which is self-referential and describes its own means of verification.

Even though it might be troubling in principle, measurement problem doesn't cause much trouble in practice. This is the subject of sections 14-18 and 22.

[23] Peres (2002), page 173

14 What is measurement?

How do we know which observables are measured? It's ambiguous, because measurement is a type of physical process that doesn't have any perfectly natural and unambiguous definition. But "river" doesn't have any perfectly natural and unambiguous definition, either, and that doesn't prevent it from being a useful and important concept. We can adopt the same attitude about measurement: it has a *useful* definition, even though it's technically ambiguous. Here's one way to say it:

> A **measurement** that distinguishes between two possibilities, x and y, is a physical process in which the rest of the system is affected in different ways by x and y, and the difference is practically irreversible.

Example: we can measure the location of an object by shining light on it and recording the reflected light with a camera. In this example, the object's location affects the pattern of light recorded by the camera, and the difference between the effects caused by different locations is practically irreversible.[24]

In everyday language, we normally reserve the word *measurement* for processes that provide useful information to people. The definition highlighted above removes that anthropic flavor: it doesn't refer to people, or to any other sentient beings. People use measurements, but measurements don't need people. Measurements as defined above are happening naturally all the time. This is part of why classical models tend to work well in macroscopic applications (section 22).

The definition highlighted above is ambiguous, partly because of the ambiguous words *practically irreversible*. This ambiguity cannot be eliminated without choosing artificial thresholds. The definition is good enough to be useful, though, because a high-quality measurement is like what happens when we pop a balloon: once the

[24] It's still practically irreversible even if the record that the camera forms is melted or pulverized, because the microscopic details of the final state of the rubble (and of the environment) still depend on the original state of the record.

process is initiated, it progresses so rapidly and so prolifically that we could not reverse it even if we wanted to. For most practical purposes, the difference between popping and not popping a balloon is unambiguous.

The principles in section 7 directly apply only to high-quality measurements that resolve all of the possible outcomes from each other with essentially no overlap. These are often called **sharp** measurements. Many realistic measurements are not sharp, because they have limited resolution, but the principles in section 7 are still sufficient if we treat unsharp measurements as *indirect* measurements. This works because the result of an unsharp measurement is typically "read out" using another measurement (such as by observing a digital display) that has only a finite set of easily-distinguishable possible outcomes. The principles in section 7 can be applied to this sharp measurement.

15 Accounting for measurement: two approaches

Measurement is a physical process. The act of measurement has consequences, even if we ignore the outcome. We can account for this in either of two ways:

- **The natural approach:**[25] We can use a model that encompasses the complex microscopic details of the physical measurement equipment, and we can use a state that accounts for the initial physical configuration of those things, all as part of one large closed quantum system. Then the measurement is a physical process that occurs within the model itself, just like any other physical process within the model's scope.

- **The artificial approach:**[26] We use a model or state that is too simplistic to describe measurement as a physical process.[27] When a measurement occurs in the real world, we modify the state by hand.

In both approaches, quantum theory doesn't predict which measurement outcome we will actually experience (except when Born's rule assigns a probability very close to 1 to one of these outcomes), so we still need to use the state-update rule to account for that. One advantage of the natural approach is that if we don't want to condition future predictions on the outcome of one measurement, then we don't need to use the state-update rule for that measurement. We can treat that measurement as a physical process to be studied via later measurements, applying the state-update rule to them instead. Another advantage of the natural approach is that even when we do use the state-update rule, we don't need to rely on any unmodeled information to tell us when a measurement has occurred: we can diagnose the occurrence of a measurement within the model itself, using a principle that will be described in section 17. The disadvantage of the natural approach is that it is usually too difficult. To make calculations easier, we normally use the artificial approach instead.

[25] Sections 16-18

[26] Sections 19-21

[27] Few-particle models, like the ones often used to introduce quantum theory, are in this category.

16 Observables and complexity

Section 17 will explain how the definition of measurement in section 14 can be formulated mathematically when the natural approach is used. That formulation relies on the idea that many of the operators that we might designate as *observables* are not practically measurable, because they are too complex. We might imagine doing a double-slit interference experiment using the whole earth as the "particle," but such an experiment will never actually be feasible.[28]

Conversely, if an operator O represents an observable that is simple enough to measure in practice, then O cannot mix two states that differ from each other in very complicated ways (as quantified by other observables associated with the same time as O).[29] More precisely: if two state-vectors $|1\rangle$ and $|2\rangle$ represent configurations that differ from each other in very complicated ways according to observables at time t, and if Ω is the set of operators representing observables that we could realistically measure at time t, then the inner product of $P|1\rangle$ and $P|2\rangle$ is practically zero (compared to the norms of $|1\rangle$ and $|2\rangle$) whenever P is a projection operator from Ω.

Many researchers have used this too-complex-to-measure idea as an input, at least implicitly, even though very few have quantified the idea itself.[30] I'll do the same thing in section 17: I'll use the idea as an input, but I won't try to quantify the idea or to deduce it from any other principles.

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[28] Omnès (1994) describes a quantitative example on pages 308-309 to explain why the word *never* is justified: such an experiment would require more resources than the entire known universe can provide. Aaronson *et al* (2020) derives a general theorem that relates the difficulty of detecting interference between two states to the difficulty of transforming one state to the other. This theorem doesn't preclude experiments like the one reported in Bild *et al* (2023), because in that experiment, the two terms in the superposition are easy to transform into each other.

[29] Remember that we're using the Heisenberg picture, so the "same" observable is represented by different operators at different times, and the same operator can represent different observables at different times.

[30] It might start to get more quantitative attention because of its role(s) in quantum gravity research. Papers that mention such a connection include Susskind (2013) (in section 2.3) and Balasubramanian *et al* (2005). Further ideas about the role(s) of complexity in quantum gravity are highlighted in Susskind (2018) and Susskind (2019).

17 Recognizing the occurrence of a measurement

This section explains how the definition of measurement in section 14 can be formulated mathematically when the natural approach is used (section 15). Let

$$A \equiv \{P_1,\, P_2,\, ...,\, P_N\} \tag{12}$$

be a set of mutually orthogonal projection operators representing an observable that could be measured at time $t = 0$. Whether it will actually be measured depends on what initial state we choose.[31]

The occurrence of the measurement shouldn't depend on the detail to which the observable A is sensitive, because the measurement is supposed to resolve that detail. To determine whether A is measured, we need to compare what happens for N different initial states, differing from each other only in that detail. To do this, consider a collection of state-vectors $|k\rangle$ satisfying

$$P_k|k\rangle = |k\rangle, \tag{13}$$

which implies $\langle j|k\rangle = 0$ whenever $j \neq k$, because the P_ks are orthogonal to each other. For convenience, take them to be unit vectors:

$$\langle k|k\rangle = 1. \tag{14}$$

The key is to suppose that they all represent *almost* the same physical situation as far as observables associated with times $t \leq 0$ can tell, except for the detail to which the observable A is sensitive.[32]

[31] It might not be measured if the initial state represents a configuration in which the measuring equipment is absent or broken!

[32] For this to make sense, the subspaces $P_n\mathcal{H}$ of the Hilbert space $\mathcal{H}$ must each be many-dimensional. (In a model like QED+QCD, they are all infinite-dimensional.) These are the observable's **eigenspaces**. Observables whose eigenspaces are one-dimensional might be reasonable in simple models that *don't* include things made of jillions of molecules, but measuring an "observable" with one-dimensional eigenspaces is not feasible in any model that is comprehensive enough to include things made of jillions of molecules.

What is the effect of measuring the observable A? Let Ω be the set of operators representing observables that we could realistically measure at times after the A-measurement event.[33,34] Saying that A has been measured means that the feature to which A is sensitive has left a practically irreversible imprint on the rest of the system, so that the N vectors $|k\rangle$ all differ from each other in hopelessly complicated ways with respect to observables at times after the measurement event.[35] According to the idea reviewed in section 16, this means

$$\left|\langle j|Q|k\rangle\right|^2 \lll 1 \qquad \text{for } j \neq k \tag{15}$$

for all projection operators Q in Ω.[36] This is the condition that the observable (12) should satisfy if it has been measured.[37] The quality of the approximation (the degree to which $\langle j|Q|k\rangle$ is negligible when $j \neq k$) reflects the quality of the measurement.

Now consider a state of the form (8) where $|\psi\rangle$ is a **superposition** (linear combination) of the vectors $|k\rangle$:

$$|\psi\rangle = \sum_k z_k|k\rangle. \tag{16}$$

The condition (15) implies[38]

$$\left|\rho(P_jQP_k)\right|^2 \lll \rho(P_j)\rho(P_k) \qquad \text{for } j \neq k \tag{17}$$

[33]Real measurements have a finite duration, but the duration may be very short, even for non-deliberate measurements. This is quantified in Zurek (2003) and Tegmark (2000).

[34]Remember that we're using the Heisenberg picture: observables at different times are represented by different operators.

[35]Page 74 in Zeh (1970), Peres (1980), Lloyd (1988), section 3 in Banks *et al* (2002), Banks (2009)

[36]Instead of imposing this condition only for individual projection operators, we could impose it for time-ordered sequences of projection operators (replacing Q with M^*M, where M is a time-ordered sequence of projection operators from Ω, as in section 10), but the single-projection version is simpler and probably strong enough already.

[37]Bell (2004) describes a simple (but contrived) example, emphasizing that (15) can only hold for a limited set Ω of observables.

[38]The condition $|x|^2 \lll 0$ is understood to mean $x = 0$.

for all states of the form (16), for all projection operators $Q \in \Omega$. We can use this as a more general way of expressing the definition of measurement, for states that are not necessarily of the form (8), but we need to be careful: checking the condition (17) for a single state ρ is not enough.[39] The definition of *measurement* involves a comparison. We need to consider at least N states that differ from each other in the detail to which the observable (12) is sensitive, because we need to determine whether the rest of the system is affected in different ways by different configurations of that detail.

This formulation is more explicit than the verbal definition that was highlighted in section 14, but it's still ambiguous: the approximations are unquantified, and the scope of Ω is only vaguely defined. In models that use a finite-dimensional Hilbert space,[40] this formulation also has another ambiguity, because then the **Poincaré recurrence theorem** says that history must eventually essentially repeat itself.[41] That creates an ambiguity because the definition of Ω refers to observables in the *future* of the A-measurement event. If Ω also included all realistically measurable observables from the *past* of the A-measurement event, then the criterion (15) would be too strong – it would not be satisfiable. To use the criterion (15) in such a model, we need to choose yet another artificial threshold to specify how far into the future the set of "future" observables Ω should go, so they don't wrap around into the (recent) "past." This is another facet of the measurement problem that was highlighted in section 13.

In spite of those ambiguities, the formulation described in this section can help us understand why quantum theory is so robust. The next section explains how.

[39] To see this, suppose $\rho(\cdots) = \chi(\cdots|P_1)$ for an arbitrary state χ. This one state ρ satisfies the condition (17), but for a trivial reason: the projection operators P_k are mutually orthogonal.

[40] Banks (2001) indicates that if the universe is asymptotically like de Sitter spacetime (which seems to be the case), then the Hilbert space should be finite-dimensional.

[41] Banks *et al* (2002), Bousso and Susskind (2012)

18 A consequence of measurement

Measurement is a physical process. The act of measurement has consequences, even if we ignore the measurement's outcome – that is, even if we don't use the state-update rule to tell the theory which outcome we experienced. In the natural approach, those consequences occur automatically, within the model itself. We don't need to put them in by hand.[42]

To see how this works, start with equation (17), which may also be written $\left|\rho(P_j Q P_k)\right|^2 \approx 0$. This implies

$$\sum_{j,k} \rho(P_j Q P_k) \approx \sum_k \rho(P_k Q P_k) \tag{18}$$

for all $Q \in \Omega$. The left-hand side is equal to $\rho(Q)$, and the right-hand side is equal to $\sum_k \rho(Q|P_k)\rho(P_k)$, so equation (18) may also be written

$$\rho(Q) \approx \sum_k \rho(Q|P_k)\rho(P_k) \tag{19}$$

for all $Q \in \Omega$. This is a nontrivial condition: for most projection operators and most states, the left- and right-hand sides of (19) are not equal to each other, not even approximately.[43] In the natural approach, equation (19) is a necessary condition for the observable (12) to qualify as having been measured.

To appreciate the significance of the condition (19), recall **Bayes' theorem**. Bayes' theorem is a consequence of the standard rules for manipulating probabilities.[44] It says that if $p(X)$ is the probability assigned to an event X, and if $p(Y|X)$ is the probability assigned to another event Y conditioned on the occurrence of X, then

$$p(Y) = \sum_n p(Y|X_n)p(X_n) \tag{20}$$

[42] In the artificial approach (section 19), we do need to put them in by hand.

[43] Example: choose vectors $|n\rangle$ so that $P_n|n\rangle = |n\rangle$ and $\langle n|n\rangle = 1$. Take Q to be the projection onto $|q\rangle \equiv |1\rangle + |2\rangle$, and take $\rho(\cdots) = \langle q|\cdots|q\rangle/\langle q|q\rangle$. Then the left-hand side of (15) equals 1, but the right-hand side equals 1/2.

[44] Bayes' theorem is a consequence of what Jaynes (2003) calls the **product rule**.

for any set of mutually exclusive events X_n with $\sum_n p(X_n) = 1$. The principles introduced in section 7 relate this to equation (19). Starting with a state ρ in which the observable (12) will be measured, the principles section 7 say that $\rho(P_n)$ is the probability of outcome P_n and that $\rho(Q|P_n)$ is the probability of Q conditioned on the occurrence of P_n. According to Bayes' theorem, if we don't condition on the outcome of the first measurement (the measurement of the observable (12)), then the probability assigned to Q for the second measurement should be given by the right-hand side of (19). Equation (19) says that this is practically equal to $\rho(Q)$, which is what we would have used as the probability of outcome Q according to Born's rule if we didn't know that the observable (12) had been measured.

The important message here is that in the natural approach, the model itself knows that a measurement occurred, even if we don't. More importantly, in cases where the "measurement" has such low quality that we're not even sure we should call it a measurement, the model itself still properly accounts for its consequences, however messy or difficult to characterize they might be, regardless of where we might arbitrarily choose to draw the line between "measurement" and "non-measurement."

The natural approach doesn't solve the measurement problem, though. Even in the natural approach, we still need to use the state-update rule if we want to condition future predictions on the outcome of a previous measurement. One conceptual advantage of the natural approach is that we only need to do this for sharp measurements, where the line is practically unambiguous, because the model itself automatically accounts for how those outcomes are affected by any intervening unsharp measurements. This helps explain why quantum theory is so robust in spite of ambiguities in the definition of measurement.

19 Sharp measurements in the artificial approach

Measuring something and then ignoring the outcome is different than not measuring it at all. In the natural approach, the model itself knows the difference (section 18). However, that requires using a model and a state that accounts for the microscopic complexities of macroscopic objects like measuring devices.[45] If we're not doing that, then we need to use the artificial approach instead.

For sharp measurements, the artificial approach is a straightforward application of Bayes' theorem. Consider a sequence of two measurements. The first one has possible outcomes represented by projection operators $\{P_1, P_2, ...\}$. Another projection operator Q represents a possible outcome of a second measurement. Starting with the state ρ, the principles introduced in section 7 say that $\rho(P_n)$ is the probability of outcome P_n and that $\rho(Q|P_n)$ is the probability of Q conditioned on the occurrence of P_n. According to Bayes' theorem, if we don't condition on the outcome of the first measurement, then the probability assigned to Q should be

$$\sum_n \rho(Q|P_n)\rho(P_n). \tag{21}$$

In the natural approach, this is practically equal to $\rho(Q)$ (equation (19)). If we're using a model or state that doesn't account for the physical process of measurement, then the quantity (21) is generally not equal to $\rho(Q)$, not even approximately,[46] but we can account for the occurrence of the first measurement artificially by replacing the original state $\rho(\cdots)$ with

$$\rho'(\cdots) \equiv \sum_n \rho(\cdots|P_n)\rho(P_n) \tag{22}$$

to enforce Bayes' theorem, as in section 11.

[45] We often use simpler models that don't have such states (footnote 27). A quantum field theory that includes both electrons and quarks interacting via the electromagnetic and strong interactions is one example of a model that does have such states, but explicitly constructing a state that describes macroscopic measuring devices in that model is beyond our current abilities.

[46] Footnote 43 in section 18

20 Unsharp measurements in the artificial approach

To handle unsharp measurements in the artificial approach, we have two options. One option is to treat the unsharp measurement as a sharp measurement of a coarser (less discriminating) observable. Suppose the original observable is represented by three projection operators $\{P_1, P_2, P_3\}$. These are mutually orthogonal (section 2), so the partial sum

$$P_{12} \equiv P_1 + P_2$$

is still a projection operator. A sharp measurement of the coarser observable $\{P_{12}, P_3\}$ can be interpreted as a measurement of the original observable that fails to distinguish between P_1 and P_2. We can account for this by replacing

$$\rho(\cdots) \to \rho(\cdots|P_{12})\rho(P_{12}) + \rho(\cdots|P_3)\rho(P_3),$$

which is just (22) applied to the coarser observable.[47] Most unsharp measurements are not like this, though: they are unsharp in smoother ways that can't be emulated by a sharp measurement of a coarser observable.

To motivate a better approach, consider the effect of an indirect measurement in a larger model, using the natural approach. To organize the explanation, let $S(t)$ denote the set of observables associated with the subsystem of interest (example: a single atom) at time t, and let $R(t)$ denote the set of observables associated with the *rest* of the larger system at time t, which includes any measuring devices. An unsharp measurement of an observable in $S(0)$ can be treated as a physical process whose effects can be probed using sharp measurements of observables in $R(t)$ at some later time $t > 0$, like digital readouts. Let $Q_1, Q_2, ...$ be the projection operators representing the possible outcomes of one of these later measurements. If the outcome Q_n is obtained, then we account for this by replacing

$$\rho(\cdots) \to \rho(\cdots|Q_n), \tag{23}$$

[47] Remember that $\rho(\cdots|P_{12})\rho(P_{12})$ is typically *not* equal to $\rho(\cdots|P_1)\rho(P_1) + \rho(\cdots|P_2)\rho(P_2)$.

where ρ is the initial state, as in section 19. The projection operators Q_n belong to $R(t)$, not $S(t)$, but the algebra generated by $S(t)$ may include operators M_n that satisfy[48]

$$\rho(Q_n A Q_n) \approx \rho(M_n^* A M_n) \qquad \text{for all } A \in S(t) \tag{24}$$

to a good approximation. The operators M_n are not necessarily projection operators, but they do satisfy this generalization of (2):[49]

$$\sum_n M_n^* M_n = 1. \tag{25}$$

When such operators exist, equation (24) implies that as far as the subsystem S is concerned, the effect of the replacement (23) can emulated by the replacement

$$\rho(\cdots) \to \rho(\cdots | M_n). \tag{26}$$

If we don't want to condition on the outcome, then we use the replacement

$$\rho(\cdots) \to \sum_n \rho(\cdots | M_n)\rho(M_n^* M_n) \tag{27}$$

instead. This generalizes (22).

Unlike the right-hand side of (23), the right-hand side of (26) involves only operators associated with the subsystem S, so we can use (26) in a smaller model that doesn't include the rest of the larger system.[50]

[48] Preskill (2022) (summarized by equation 1.2 in Stelmachovic and Buzek (2001)), section II.D in Peres and Terno (2004), section 4.1 in Vinjanampathy and Anders (2016), and Wolf (2012)

[49] Set $A = 1$ in (24) and use $\sum_n Q_n = 1$ to deduce $\sum_n \rho(M_n^* M_n) = 1$. If this were required to hold for all states ρ, then it would imply (25). The state on the left-hand side of (24) is *not* arbitrary, because only in some states does the measurement event actually occur. (That's the point of the natural approach: measurement is a physical process whose occurrence is contingent on the initial state, just like any other physical process.) However, on the right-hand side of (24), the state is arbitrary as far as operators in the algebra generated by $S(t)$ are concerned, because the occurrence of the measurement is not contingent on the state of the subsystem of interest. This implies (25).

[50] Beware that the time-dependence of $S(t)$ in the larger model is different than in the subsystem-only model, because in the larger model, time evolution mixes the subsystem's observables with observables from the rest of the system.

21 Generalized measurement

The operators M_n in equations (25)-(27) are called **Kraus operators**, and the replacement (26) is called a **generalized measurement**. It reduces to the state-update rule (section 7) when the Kraus operators are projection operators. The terms $M_n^* M_n$ in the sum (25) are positive operators (article 74088), and they are collectively called a **positive operator valued measure (POVM)**.

Introductions to generalized measurements often highlight something called **Naimark's dilation theorem**, which says that any generalized measurement can be represented as a projective measurement in an enlarged system.[51] However, the enlarged system promised by Naimark's theorem is not meant to be (and usually isn't) a good model of any *real* larger system in which measurement can be described as a physical process. Such a good model may exist, but Naimark's theorem doesn't tell us how to find it, so Naimark's theorem is not directly relevant to the discussion in section 20. Introductions to Naimark's theorem can still be enlightening, though, because they often include simple examples of how generalized measurements can arise from projective measurements on a larger system, realistic or not.[52]

The important message in section 20 is that the generalization (26) is a result, not a postulate. It's a way of compensating for the deficiencies of a model that only knows about one relatively simple part of a complex physical system. We use such deficient models to make the math easier. In principle, we could the natural approach instead (section 15) – we could use a more comprehensive model in which unsharp measurements can be described as physical processes whose consequences are probed using subsequent sharp measurements, for which the rules in section 7 are sufficient.

[51] Peres (2002), section 9-6

[52] Footnote 48 in section 20

22 Why classical models often work well

Measurements as defined in section 14 are happening naturally all the time.[53] The location of a macroscopic object is constantly being measured through its influence on things like the ambient air, on the light from distant stars, and on the cosmic background radiation.[54] Even though observables are not all compatible with each other, any two observables can be simultaneously measured with sufficiently low quality.[55] Relationships like the **uncertainty principle** can be used to quantify how much the quality must be compromised to make the measurements compatible with each other. Naturally occurring measurements tend to have coarse enough resolution to be practically compatible with each other, even if the resolution is still fine by macroscopic standards. This helps explain why classical models – models in which all observables are compatible with each other – tend to be good approximations for most macroscopic applications.

In particular, if Δx and Δp are the resolutions of measurements of an object's location and momentum, then those measurements can be practically compatible with each other if they satisfy $(\Delta x)(\Delta p) \gg \hbar$, even though they would not be compatible with each other if $(\Delta x)(\Delta p) \lesssim \hbar$.[56] The location of a mote of dust of size 10^{-5} meter is constantly measured through its influence on the cosmic microwave background, and Tegmark (1993) finds that the resolution of this measurement is $\Delta x \sim 10^{-8}$ meter. That resolution may be fine by macroscopic standards, but it's coarse enough to be compatible with a momentum measurement whose resolution Δp is also very fine by macroscopic standards.

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[53] Sometimes the word *measurement* is reserved for deliberate measurements, I'm using the word for any physical process that has the characteristics highlighted in section 14.

[54] This is studied quantitatively in Joos and Zeh (1985) and reviewed in Joos (1996), section 3.2. Schlosshauer (2019) gives a more recent review with an extensive bibliography.

[55] An example is described in von Neumann (1955), section V-4, pages 402-404.

[56] This comes from the fact that the operator P corresponding to a given component of the system's total momentum is the generator of spatial translations in that direction. More explicitly: $U(x) = \exp(-iPx/\hbar)$ is the unitary operator that generates overall translations in space, so if a system described by a state-vector $|\psi\rangle$ has features of size $\sim \Delta x$, in the sense that the vectors $|\psi\rangle$ and $U(\Delta x)|\psi\rangle$ are essentially orthogonal to each other, then its momentum cannot be more sharply defined than $\Delta p \sim \hbar/\Delta x$.

23 The significance of commuting observables

Section 9 considered two observables that don't share any eigenvectors. Two such observables cannot commute with each other.[57,58] This section considers the opposite extreme – two observables that do commute with each other.

Let A and B be two observables, each represented by a list of mutually orthogonal projection operators:

$$A = \{P_1, P_2, ...\} \qquad\qquad B = \{Q_1, Q_2, ...\}. \tag{28}$$

Consider the condition

$$\sum_n \rho(Q_k|P_n)\rho(P_n) = \rho(Q_k) \qquad\qquad \text{for all states } \rho. \tag{29}$$

In words, this says that the occurrence of an A-measurement cannot affect the distribution of outcomes of a B-measurement if we don't condition on the outcome of the A-measurement.

The condition (29) holds if and only if A and B commute with each other – that is, if and only if all of the P_ns commute with all of the Q_ns. To prove this, first use the definition of $\rho(\cdots|P_n)$ and $P_n^* = P_n$ to see that the condition (29) can also be written

$$\sum_n \rho(P_n Q_k P_n) = \rho(Q_k) \qquad\qquad \text{for all states } \rho. \tag{30}$$

To prove that commutativity implies (30), use commutativity and $P_n^2 = P_n$ to get $\rho(P_nQ_kP_n) = \rho(Q_kP_n)$, and then use $\sum_n P_n = 1$ (as required by (2)) to get (30). To prove that (30) implies commutativity, use the fact that if (30) holds for all

[57]Recall that two operators X and Y are said to **commute** with each other if the order in which they are multiplied does not matter: $XY = YX$.

[58]The converse is false, because two operators may share an eigenvector even if they don't commute with each other.

states ρ, then it implies the operator equation

$$\sum_n P_n Q_k P_n = Q_k.$$

Multiplying this equation on the left or right by P_j and using (3) gives either $P_j Q_k P_j = P_j Q_k$ or $P_j Q_k P_j = Q_k P_j$, respectively, and comparing these two results gives $P_j Q_k = Q_k P_j$, so the the Ps and Qs must commute with each other.

Altogether, this shows that the two observables (28) satisfy (29) if and only if they commute with each other. In relativistic quantum field theory (QFT), observables associated with regions of spacetime that cannot connected to each other by any causal worldline[59] are required to commute with each other, a principle called **microcausality**.[60] That principle is meant to exclude faster-than-light influences,[61] but the result derived in this section is only part of that story. The recognition that some so-called *observables* are not actually measurable[62] is another part of the story. Article 41818 explains why that matters.

[59] A worldline is called **causal** if it is not spacelike anywhere (article 48968).

[60] Article 21916

[61] Relativistic QFT is consistent with the experimental fact that Bell inequalities are violated (article 70833), which does not require any such influences.

[62] Section 16

24 Can Born's rule be derived?

Born's rule uses a state (a normalized positive linear functional) to assign probabilities to projection operators representing the possible outcomes of a measurement. In quantum theory, Born's rule is a *postulate*: it is not derived from anything else.[63]

Gleason's theorem and related theorems (article 77228) say that Born's rule is the only option[64] if we require

$$\rho(P+Q) = \rho(P) + \rho(Q) \qquad \text{whenever} \qquad PQ = 0 \tag{31}$$

for all observables.[65] Those theorems provide valuable insight, but they don't quite qualify as derivations of Born's rule, because they don't care which observables will actually be measured. Assigning a nonzero probability to an outcome that we know will not occur – because we know that the observable will not be measured at all – is not necessary. If we only require (31) for observables that will actually be measured, then Gleason's theorem and its cousins do not apply, because they rely on the stronger premise that (31) holds for *all* observables.

Any attempt to derive Born's rule from the rest of quantum theory faces a similar dilemma. The derivation should only require assigning probabilities to the possible outcomes of observables that will actually be measured, so the derivation must rely on a mathematically precise criterion for determining which observables will actually be measured. Quantum theory in its current form does not provide such a criterion, at least not a precise one: the criterion described in section 17 is technically ambiguous, even if it is clear enough for practical purposes.

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[63] Caves and Schack (2005) give a mathematically thorough explanation of why one early attempt to derive Born's rule is not valid, namely the one in Hartle (1967). Section II in Farhi *et al* (1989) gives a concise explanation of how to evaluate Hartle's quantity (10) for finite N.

[64] Actually, it doesn't *quite* say this, because Born's rule isn't just a mathematical statement. It's a postulate about how the mathematical formalism relates to the real world. Any alleged derivation of Born's rule must therefore start with other postulate(s) about how the mathematical formalism relates to the real world. Merely showing that Born's rule is the only option consistent with a probability interpretation would not be enough to demonstrate that the probability interpretation really is the correct interpretation.

[65] More precisely: for all projection operators in the von Neumann algebra that is generated by all of the model's observables (article 74088).

25 Is quantum theory the final word?

The principles of quantum theory are the foundation for the most accurate and comprehensive models we have ever had. Some quantum models even reproduce general relativity as a good approximation.[66] The most well-understood examples produce spacetimes that are asymptotically like either flat spacetime or anti de Sitter (AdS) spacetime, but the real universe appears to be asymptotically like de Sitter (dS) spacetime instead. We don't yet have any widely-accepted model that reproduces general relativity as a good approximation with this asymptotic condition.[67] Some authors have speculated that this might indicate a limitation of quantum theory, but we'll see.

Historically, most discussions about the adequacy of quantum theory have revolved around the measurement problem. Some authors have suggested that the measurement problem might not really be a problem,[68] and some have suggested that it might be a symptom that quantum theory is not the final word.[69] In any case, no compelling alternatives to quantum theory have been substantiated yet.

Quantum theory may or may not be the final word,[70] but one thing is practically certain: it will always be an important part of the physics curriculum. We know now that Newton's model of gravity was not the final word, but it still works just as well as it worked originally. General relativity is even better, but Newton's model is easier and often sufficient, so Newton's model is still taught first. Quantum theory will always be an important tool, too, even if it isn't the final word.

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[66] Examples include the **BFSS M(atrix) model**, proposed in Banks *et al* (1997) and reviewed in Banks (1998) and Bigatti and Susskind (1997), and the **AdS/CFT correspondence**, which is reviewed in Polchinski (2010), Maldacena (2011), Sundrum (2012), Hubeny (2015), Kaplan (2016), and Van Raamsdonk (2017). These models allegedly provide nonperturbative definitions of string theory in spacetimes that are asymptotically flat and asymptotically AdS, respectively.

[67] Papers on this topic include Witten (2001), Strominger (2001), Banks (2015), and Susskind (2021).

[68] One example is Coleman (1994). This perspective can be traced back to Everett (1957), which introduced what is often called **many-worlds interpretation** (MWI) of quantum theory. It doesn't avoid the need for a criterion like (15), but it does provide a nice perspective for thinking about such criteria, as explained in section 17.

[69] Weinberg (2013), page 95

[70] This is occasionally mentioned in quantum gravity research (example: Bousso (2002), text below equation (8.1)).

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